Dawson College: Calculus II: 201-NYB-05 05

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Last Name: SOLUTIONS

First Name: ______________

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Quiz 11 (A)

Question 1. Determine whether or not the following series are absolutely convergent, conditionally convergent or divergent.

(a) *(4 marks)*

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(\arctan n)^n}$$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(-1)^n}{(\arctan n)^n}\right|} = \lim_{n\to\infty} \frac{1}{\arctan n} = \frac{1}{\pi/2} = \frac{2}{\pi} < 1$$

$\therefore$ THE SERIES CONVERGES ABSOLUTELY BY ROOT TEST

(b) *(6 marks)*

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}, \qquad \sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

BUT THIS SERIES DIVERGES BY LIMIT COMPARISON TEST WITH $\sum_{n=1}^{\infty} \frac{1}{n}$ (DIVERGES, p-SERIES, $p=1$), SINCE

$$\lim_{n\to\infty} \frac{\frac{n}{n^2+1}}{1/n} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$$

SO IT ISN'T ABSOLUTELY CONVERGENT.

$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$ IS AN ALTERNATING SERIES SO

1) LET $f(x) = \frac{x}{x^2+1} \implies f'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} < 0$

FOR $x > 1$ SO $b_{n+1} < b_n$

2) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n}{n^2+1} = 0$ $\therefore$ THE SERIES CONVERGES BY ALTERNATING SERIES TEST

$\therefore$ THE SERIES IS CONDITIONALLY CONVERGENT.