

Last Name: SOLUTIONS

First Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

## Quiz 11 (B)

**Question 1.** Determine whether or not the following series are absolutely convergent, conditionally convergent or divergent.

(b) (4 marks)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(\arctan n)^n}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \sqrt[n]{\left| \frac{(-1)^n}{(\arctan n)^n} \right|} = \lim_{n \rightarrow \infty} \frac{1}{\arctan n} = \frac{1}{\pi/2} = \frac{2}{\pi} < 1$$

$\therefore$  THE SERIES CONVERGES ABSOLUTELY BY ROOT TEST

(a) (6 marks)

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}, \quad \sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

BUT THIS SERIES DIVERGES BY LIMIT COMPARISON TEST WITH  $\sum_{n=1}^{\infty} \frac{1}{n}$  (DIVERGES, P-SERIES,  $p=1$ ), SINCE

$$\lim_{n \rightarrow \infty} \frac{\frac{n}{n^2+1}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2+1} = 1$$

SO IT ISN'T ABSOLUTELY CONVERGENT.

$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$  IS AN ALTERNATING SERIES SO

$$1) \text{ LET } f(x) = \frac{x}{x^2+1} \Rightarrow f'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} < 0$$

FOR  $x > 1$  SO  $b_{n+1} < b_n$

$$2) \lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{n}{n^2+1} = 0 \quad \therefore \text{THE SERIES CONVERGES BY ALTERNATING SERIES TEST}$$

$\therefore$  THE SERIES IS CONDITIONALLY CONVERGENT.