**Dawson College: Calculus II: 201-NYB-05 05** February 4, 2011

**Last Name:** SOLUTIONS

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# Quiz 2 (A)

**Question 1.** *(10 marks)* Evaluate the following definite integrals using Riemann Sums:

$$\int_2^3 (x-x^2)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{3-2}{n} = \frac{1}{n}, \quad x_i = a + i\Delta x = 2 + i\left(\frac{1}{n}\right) = 2 + \frac{i}{n}$$

$$f(x_i) = \left(2+\frac{i}{n}\right) - \left(2+\frac{i}{n}\right)^2 = 2+\frac{i}{n} - \left(4+\frac{4i}{n}+\frac{i^2}{n^2}\right)$$

$$= 2+\frac{i}{n} - 4 - \frac{4i}{n} - \frac{i^2}{n^2} = -2 - \frac{3i}{n} - \frac{i^2}{n^2}$$

$$f(x_i)\Delta x = \left(-2-\frac{3i}{n}-\frac{i^2}{n}\right)\cdot\left(\frac{1}{n}\right) = -\frac{2}{n} - \frac{3i}{n^2} - \frac{i^2}{n^3}$$

$$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left(-\frac{2}{n} - \frac{3i}{n^2} - \frac{i^2}{n^3}\right)$$

$$= -\frac{2}{n}\sum_{i=1}^{n} 1 - \frac{3}{n^2}\sum_{i=1}^{n} i - \frac{1}{n^3}\sum_{i=1}^{n} i^2$$

$$= \frac{-2}{n}\cdot n - \frac{3}{n^2}\cdot\frac{n(n+1)}{2} - \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$

$$\therefore \int_2^3 (x-x^2)\,dx = \lim_{n\to\infty}\left[-2 - \frac{3}{2}\cdot\frac{n}{n}\cdot\frac{n+1}{n} - \frac{1}{6}\cdot\frac{n}{n}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n}\right]$$

$$= \lim_{n\to\infty}\left[-2 - \frac{3}{2}\cdot 1\cdot\left(1+\frac{1}{n}\right) - \frac{1}{6}\cdot 1\cdot\left(1+\frac{1}{n}\right)\cdot\left(2+\frac{1}{n}\right)\right]$$

$$= -2 - \frac{3}{2}\cdot(1)\cdot(1+0) - \frac{1}{6}\cdot 1\cdot(1+0)\cdot(2+0)$$

$$= -2 - \frac{3}{2} - \frac{2}{6} = \frac{-12}{6} - \frac{9}{6} - \frac{2}{6} = -\frac{23}{6}$$