Dawson College: **Calculus II**: 201-NYB-05 05

February 4, 2011

Last Name: SOLUTIONS

First Name: ____________

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Quiz 2 (B)

Question 1. *(10 marks)* Evaluate the following definite integrals using Riemann Sums:

$$\int_3^4 (x^2 - x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{4-3}{n} = \frac{1}{n}, \quad x_i = a + i\Delta x = 3 + i\left(\frac{1}{n}\right) = 3 + \frac{i}{n}$$

$$f(x_i) = \left(3+\frac{i}{n}\right)^2 - \left(3+\frac{i}{n}\right) = 9 + \frac{6i}{n} + \frac{i^2}{n^2} - 3 - \frac{i}{n} = 6 + \frac{5i}{n} + \frac{i^2}{n^2}$$

$$f(x_i)\Delta x = \left(6 + \frac{5i}{n} + \frac{i^2}{n^2}\right)\left(\frac{1}{n}\right) = \frac{6}{n} + \frac{5i}{n^2} + \frac{i^2}{n^3}$$

$$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left(\frac{6}{n} + \frac{5i}{n^2} + \frac{i^2}{n^3}\right) = \frac{6}{n}\sum_{i=1}^{n} 1 + \frac{5}{n^2}\sum_{i=1}^{n} i + \frac{1}{n^3}\sum_{i=1}^{n} i^2$$

$$= \frac{6}{n}\cdot n + \frac{5}{n^2}\cdot\frac{n(n+1)}{2} + \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$

$$\int_3^4 (x^2-x)\,dx = \lim_{n\to\infty}\left[6 + \frac{5}{2}\cdot\frac{n}{n}\cdot\frac{n+1}{n} + \frac{1}{6}\cdot\frac{n}{n}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n}\right]$$

$$= \lim_{n\to\infty}\left[6 + \frac{5}{2}\cdot 1\cdot\left(1+\frac{1}{n}\right) + \frac{1}{6}\cdot 1\cdot\left(1+\frac{1}{n}\right)\cdot\left(2+\frac{1}{n}\right)\right]$$

$$= 6 + \frac{5}{2}\cdot 1\cdot(1+0) + \frac{1}{6}\cdot 1\cdot(1+0)(2+0)$$

$$= 6 + \frac{5}{2} + \frac{1}{3} = \frac{36}{6} + \frac{15}{6} + \frac{2}{6} = \frac{53}{6}$$