

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 5 (A)

Question 1. Evaluate the following integrals (no decimals!):

(a) (5 marks)

$$\int_0^{3\pi/8} \sin^4(2t) dt = \int_0^{3\pi/8} (\sin^2(2t))^2 dt = \int_0^{3\pi/8} \left(\frac{1 - \cos 4t}{2} \right)^2 dt$$

$$= \frac{1}{4} \int_0^{3\pi/8} 1 - 2\cos 4t + \cos^2 4t dt = \frac{1}{4} \int_0^{3\pi/8} 1 - 2\cos 4t + \frac{1 + \cos 8t}{2} dt$$

$$= \frac{1}{4} \int_0^{3\pi/8} \frac{3}{2} - 2\cos 4t + \frac{1}{2} \cos 8t dt = \frac{1}{4} \left[\frac{3}{2}t - \frac{2}{4} \sin 4t + \frac{1}{16} \sin 8t \right]_0^{3\pi/8}$$

$$= \frac{3}{8} \left(\frac{3\pi}{8} \right) + \frac{2}{16} \sin \frac{3\pi}{2} + \frac{1}{64} \sin 3\pi - \frac{3}{8}(0) - \frac{2}{16} \sin(0) - \frac{1}{64} \sin(0)$$

$$= \frac{9\pi}{64} - \frac{2}{16} + 0 - 0 - 0 - 0 = \frac{9\pi}{64} - \frac{2}{16}$$

(b) (5 marks)

$$\int \tan^5 x \sec^5 x dx = \int \tan^4 x \sec^4 x \cdot \tan x \sec x dx$$

$$= \int (\sec^2 x - 1)^2 \sec^4 x \cdot \tan x \sec x dx$$

$$= \int (u^2 - 1)^2 u^4 du = \int (u^4 - 2u^2 + 1) u^4 du$$

$$= \int u^8 - 2u^6 + u^4 du = \frac{u^9}{9} - \frac{2u^7}{7} + \frac{u^5}{5} + C$$

$$= \frac{\sec^9 x}{9} - \frac{2\sec^7 x}{7} + \frac{\sec^5 x}{5} + C$$

LET
 $u = \sec x dx$
 $du = \tan x \sec x dx$