

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 5 (B)

Question 1. Evaluate the following integrals:

(a) (5 marks)

$$\int_0^{3\pi/12} \cos^4(3t) dt = \int_0^{3\pi/12} (\cos^2(3t))^2 dt = \int_0^{3\pi/12} \left(\frac{1 + \cos 6t}{2} \right)^2 dt$$

$$= \frac{1}{4} \int_0^{3\pi/12} 1 + 2\cos 6t + \cos^2 6t dt = \frac{1}{4} \int_0^{3\pi/12} 1 + 2\cos 6t + \frac{1 + \cos 12t}{2} dt$$

$$= \frac{1}{4} \int_0^{3\pi/12} \frac{3}{2} + 2\cos 6t + \frac{1}{2} \cos 12t dt = \frac{1}{4} \left[\frac{3}{2}t + \frac{2}{6} \sin 6t + \frac{1}{24} \sin 12t \right]_0^{3\pi/12}$$

$$= \frac{3}{8} \left(\frac{3\pi}{12} \right) + \frac{2}{24} \sin \frac{3\pi}{2} + \frac{1}{24} \sin 3\pi - \frac{3}{8}(0) - \frac{2}{6} \sin(0) - \frac{1}{24} \sin(0)$$

$$= \frac{3\pi}{32} - \frac{1}{12}$$

(b) (5 marks)

$$\int \tan^6 x \sec^6 x dx = \int \tan^6 x \sec^4 x \sec^2 x dx$$

$$= \int \tan^6 x (\tan^2 x + 1)^2 \sec^2 x dx = \int u^6 (u^2 + 1)^2 du$$

$$\text{Let } u = \tan x$$

$$du = \sec^2 x dx$$

$$= \int u^6 (u^4 + 2u^2 + 1) du = \int u^{10} + 2u^8 + u^6 du$$

$$= \frac{u^{11}}{11} + \frac{2u^9}{9} + \frac{u^7}{7} + C$$

$$= \frac{\tan^{11} x}{11} + \frac{2 \tan^9 x}{9} + \frac{\tan^7 x}{7} + C$$