

Last Name: SOLUTIONS

First Name: \_\_\_\_\_

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## Quiz 6 (A)

Question 1. (10 marks) Evaluate the following integral:

$$I = \int \frac{x^4 - 10x^3 + 28x^2 - 15x - 15}{x^3 - 7x^2 + 10x} dx = \int x - 3 + \frac{-3x^2 + 15x - 15}{x(x-2)(x-5)} dx$$

$$\begin{array}{r} x - 3 \\ x^3 - 7x^2 + 10x \overline{) x^4 - 10x^3 + 28x^2 - 15x - 15} \\ \underline{-(x^4 - 7x^3 + 10x^2)} \phantom{- 15} \\ -3x^3 + 18x^2 - 15x \phantom{- 15} \\ \underline{+(-3x^3 + 21x^2 - 30x)} \phantom{- 15} \\ -3x^2 + 15x - 15 \end{array}$$

$$\frac{-3x^2 + 15x - 15}{x(x-2)(x-5)} = \frac{A}{x} + \frac{B}{x-5} + \frac{C}{x-3}$$

$$-3x^2 + 15x - 15 = A(x-2)(x-5) + Bx(x-2) + Cx(x-5)$$

IF  $x=0$ 

$$-15 = 10A \Rightarrow A = -\frac{3}{2}$$

IF  $x=2$ 

$$3 = -6C \Rightarrow C = -\frac{1}{2}$$

IF  $x=5$ 

$$-75 + 75 - 15 = 15B \Rightarrow B = -1$$

$$I = \int x - 3 - \frac{3}{2} \cdot \frac{1}{x} - \frac{1}{x-5} - \frac{1}{2} \cdot \frac{1}{x-2} dx$$

$$= \frac{1}{2}x^2 - 3x - \frac{3}{2} \ln|x| - \ln|x-5| - \frac{1}{2} \ln|x-2| + C$$