

Last Name: SOLUTIONS

First Name: \_\_\_\_\_

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## Quiz 6 (B)

Question 1. (10 marks) Evaluate the following integral:

$$I = \int \frac{x^5 - 2x^4 - 7x^3 + 20x^2 - 12x + 4}{x^3 + x^2 - 6x} dx = \int x^2 - 3x + 2 + \frac{4}{x(x+3)(x-2)} dx$$

$$\begin{array}{r} x^3 + x^2 - 6x \overline{) x^5 - 2x^4 - 7x^3 + 20x^2 - 12x + 4} \\ \underline{-(x^5 + x^4 - 6x^3)} \phantom{+ 4} \\ -3x^4 - x^3 + 20x^2 \phantom{- 12x + 4} \\ \underline{-(-3x^4 - 3x^3 + 18x^2)} \\ 2x^3 + 2x^2 - 12x \phantom{+ 4} \\ \underline{-(2x^3 + 2x^2 - 12x)} \\ 0 + 4 \end{array}$$

$$\frac{4}{x(x+3)(x-2)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-2}$$

$$4 = A(x+3)(x-2) + Bx(x-2) + Cx(x+3)$$

$$\text{IF } x = 0$$

$$4 = -6A \Rightarrow A = -\frac{2}{3}$$

$$\text{IF } x = 2$$

$$4 = 10C \Rightarrow C = \frac{2}{5}$$

$$\text{IF } x = -3$$

$$4 = 15B \Rightarrow B = \frac{4}{15}$$

$$I = \int x^2 - 3x + 2 - \frac{2}{3} \cdot \frac{1}{x} + \frac{4}{15} \cdot \frac{1}{x+3} + \frac{2}{5} \cdot \frac{1}{x-2} dx$$

$$= \frac{x^3}{3} - \frac{3}{2}x^2 + 2x - \frac{2}{3} \ln|x| + \frac{4}{15} \ln|x+3| + \frac{2}{5} \ln|x-2| + C$$