Dawson College: Calculus II: 201-NYB-05 05

April 1, 2011

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Quiz 7 (A)

Question 1. *(10 marks)* Evaluate the following integrals (if they converge):

(a) $\int_0^{\infty} \frac{e^x}{e^{2x}+4}dx = \lim_{t\to\infty} \int_0^t \frac{e^x}{e^{2x}+4}dx = I$

LET $u = e^x \Rightarrow du = e^x dx$

$\therefore \int \frac{e^x}{e^{2x}+4}dx = \int \frac{du}{u^2+4} = \frac{1}{4}\int \frac{du}{\left(\frac{u}{2}\right)^2+1} = \frac{1}{2}\arctan\left(\frac{u}{2}\right)+C$

$= \frac{1}{2}\arctan\left(\frac{e^x}{2}\right)+C$

$\therefore I = \lim_{t\to\infty}\left[\frac{1}{2}\arctan\left(\frac{e^x}{2}\right)\right]_0^t = \lim_{t\to\infty}\left[\frac{1}{2}\arctan\left(\frac{e^t}{2}\right) - \frac{1}{2}\arctan\frac{1}{2}\right]$

$= \frac{1}{2}\cdot\frac{\pi}{2} - \frac{1}{2}\arctan\frac{1}{2}$

(b) $\int_{\frac{1}{2}}^1 \frac{1}{\sqrt{2x-1}}dx = \lim_{t\to\frac{1}{2}^+}\int_t^1 \frac{1}{\sqrt{2x-1}}dx$

$= \lim_{t\to\frac{1}{2}^+}\left[\frac{1}{2}\cdot 2\,(2x-1)^{1/2}\right]_t^1 = \lim_{t\to\frac{1}{2}^+}\left[(2(1)-1)^{1/2} - (2t-1)^{1/2}\right]$

$= (1)^{1/2} - (0)^{1/2}$

$= 1$