Dawson College: Calculus II: 201-NYB-05 05

April 1, 2011

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Quiz 7 (B)

Question 1. *(10 marks)* Evaluate the following integrals (if they converge):

(a) $\int_{1/3}^{1} \frac{1}{\sqrt{3x-1}}dx = \lim_{t \to \frac{1}{3}^+} \int_t^1 \frac{1}{\sqrt{3x-1}}dx$

$= \lim_{t \to \frac{1}{3}^+} \left[\frac{2}{3}(3x-1)^{1/2} \right]_t^1 = \lim_{t \to \frac{1}{3}^+} \left[\frac{2}{3}(2)^{1/2} - \frac{2}{3}(3t-1)^{1/2} \right]$

$= \frac{2}{3}(2)^{1/2} - \frac{2}{3}(0)^{1/2} = \frac{2\sqrt{2}}{3}$

(b) $\int_0^{\infty} \frac{e^x}{e^{2x}+9}dx = \lim_{t \to \infty} \int_0^t \frac{e^x}{e^{2x}+9}dx = I$

LET $u = e^x \Rightarrow du = e^x dx$

$\therefore \int \frac{e^x}{e^{2x}+9}dx = \int \frac{du}{u^2+9} = \frac{1}{9}\int \frac{du}{\left(\frac{u}{3}\right)^2+1} = \frac{1}{3}\arctan\left(\frac{u}{3}\right) + C$

$= \frac{1}{3}\arctan\left(\frac{e^x}{3}\right) + C$

$\therefore I = \lim_{t \to \infty} \left[\frac{1}{3}\arctan\left(\frac{e^x}{3}\right) \right]_0^t = \lim_{t \to \infty} \left[\frac{1}{3}\arctan\left(\frac{e^t}{3}\right) - \frac{1}{3}\arctan\frac{1}{3} \right]$

$= \frac{1}{3} \cdot \frac{\pi}{2} - \frac{1}{3}\arctan\frac{1}{3}.$