Dawson College: Calculus II: 201-NYB-05 05

April 15, 2011

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 9 (A)

Question 1. *(5 marks)* Using the "cylindrical shells" set up an integral that gives the volume of the solid generated by rotating the region enclosed by $x+y=3$ and $x=4-(y-1)^2$ about the x-axis. Don't integrate this integral.

INTERSECTION:

$x+y=3 \Rightarrow x=3-y$

$4-(y-1)^2 = 3-y$

$4-(y^2-2y+1) = 3-y$

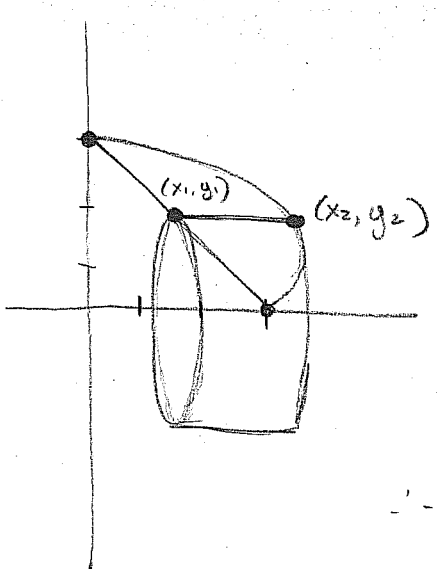
$4-y^2+2y-1 = 3-y$

$0 = y^2-3y$

$0 = y(y-3)$

$\therefore y = 0, 3$

$\therefore (3,0), (0,3)$



$$V = \int_0^3 A(y)\,dy$$

$A(y) = 2\pi r h$

$r = y$

$h = x_2 - x_1$

$= 4-(y-1)^2-(3-y)$

$= 1+y-(y-1)^2$

$$\therefore V = \int_0^3 2\pi y\left[1+y-(y-1)^2\right]dy$$

Question 2. *(5 marks)* Find the length of the curve $y=\ln(\cos x)$, from $(0,0)$ to $(\frac{\pi}{6}, \ln\frac{\sqrt{3}}{2})$.

$\frac{dy}{dx} = \frac{1}{\cos x}\cdot(-\sin x) = -\tan x \Rightarrow 1+\left(\frac{dy}{dx}\right)^2 = 1+\tan^2 x = \sec^2 x$

$$\therefore L = \int_0^{\pi/6}\sqrt{\sec^2 x}\,dx = \int_0^{\pi/6}|\sec x|\,dx = \int_0^{\pi/6}\sec x\,dx$$

since $\sec x \geq 0$ on $[0, \pi/6]$

$$= \ln|\sec x+\tan x|\Big|_0^{\pi/6} = \ln\left|\frac{2}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right| - \ln|1+0|$$

$$= \ln\left(\frac{3}{\sqrt{3}}\right)$$