Dawson College: Calculus II: 201-NYB-05 05

April 15, 2011

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 9 (B)

Question 1. *(5 marks)* Find the length of the curve $y = \ln(\cos x)$, from $(0,0)$ to $(\frac{\pi}{3}, \ln\frac{1}{2})$.

$\frac{dy}{dx} = \frac{1}{\cos x} \cdot (-\sin x) = -\tan x \Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \tan^2 x = \sec^2 x$

$\therefore L = \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/3} |\sec x|\, dx = \int_0^{\pi/3} \sec x\, dx$

since $\sec x \geq 0$ on $[0, \pi/3]$

$= \ln|\sec x + \tan x| \Big|_0^{\pi/3} = \ln|2 + \sqrt{3}| - \ln|1 + 0|$

$= \ln(2 + \sqrt{3})$

Question 2. *(5 marks)* Using the "cylindrical shells" set up an integral that gives the volume of the solid generated by rotating the region enclosed by $x + y = 3$ and $x = 4 - (y-1)^2$ about the x-axis. Don't integrate this integral.

INTERSECTION:

$x + y = 3 \Rightarrow x = 3 - y$

$\therefore 4 - (y-1)^2 = 3 - y$

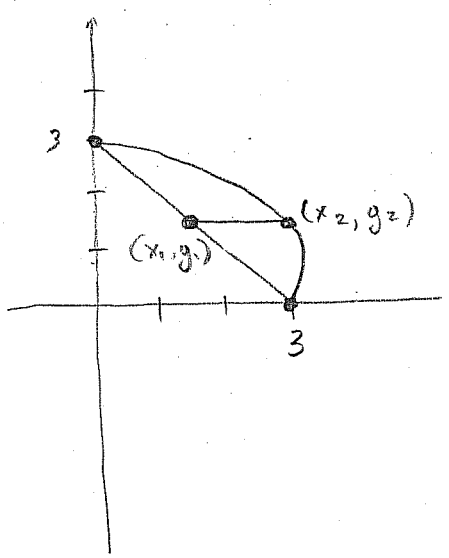
$4 - y^2 + 2y - 1 = 3 - y$

$0 = y^2 - 3y$

$0 = y(y-3)$

$\therefore y = 0, 3$

$\therefore (3,0), (0,3)$



$V = \int_0^3 A(y)\, dy$

$A(y) = 2\pi r h$

$r = y$

$h = x_2 - x_1$

$= 4 - (y-1)^2 - (3-y)$

$= 3y - y^2$

$\therefore V = \int_0^3 2\pi y(3y - y^2)\, dy$