

Last Name: _____

First Name: _____

Student ID: _____

Test 2 (A)

Question 1. (5 marks). Evaluate the indefinite integral.

$$\int_0^1 \arctan x \, dx$$

Question 2. (3 marks). Evaluate the indefinite integral.

$$\int \sin^2 3x \, dx$$

Question 3. (5 marks).

(a) Prove the reduction formula

$$\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$$

(b) Use this formula to evaluate

$$\int (\ln x)^3 dx$$

Question 4. (5 marks) Evaluate the following integral.

$$\int \frac{1}{x^2 \sqrt{9-x^2}} dx$$

Question 5. (3 marks) Write the partial fraction decomposition of the following fraction. Do not determine the numerical values of the coefficients.

$$\frac{3x+9}{x^2(2x+1)^3(x^2+1)^2}$$

Question 6. (5 marks) Evaluate the following integral.

$$\int_{3\pi/4}^{\pi} \sec^6 x dx$$

Question 7. (2 marks) Evaluate the following integral.

$$\int_{-\pi/4}^{\pi/4} \frac{\sin x \cos x}{x^4 + 3x^2 + 1} dx$$

Question 8. (5 marks) Evaluate the following integral.

$$\int \frac{x^4 + 6x^2 - 20x + 1}{x^2 + 2x + 10} dx$$

Question 9. (5 marks) Evaluate the following integral.

$$\int \frac{x^2 + x - 18}{x^3 + 9x} dx$$

Question 10. (3 marks). If f is continuous and

$$\int_1^3 f(x)dx = 36 \quad \text{find} \quad \int_0^1 x^2 f(2x^3 + 1)dx.$$

Clearly show your work.

Bonus. (3 marks) Find

$$\int \frac{1}{x^4+1} dx$$

given that

$$\frac{1}{x^4+1} = \frac{\frac{1}{2\sqrt{2}}x + \frac{1}{2}}{x^2 + \sqrt{2}x + 1} + \frac{-\frac{1}{2\sqrt{2}}x + \frac{1}{2}}{x^2 - \sqrt{2}x + 1}$$