

Last Name: \_\_\_\_\_

First Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

## Test 2 (B)

**Question 1.** (3 marks). Evaluate the indefinite integral.

$$\int \cos^2 5x \, dx$$

**Question 2.** (5 marks). Evaluate the indefinite integral.

$$\int_0^1 \arctan x \, dx$$

**Question 3.** (3 marks) Write the partial fraction decomposition of the following fraction. Do not determine the numerical values of the coefficients.

$$\frac{5x - 1}{x^2(5x + 2)^3(x^2 + 3)^2}$$

**Question 4.** (5 marks) Evaluate the following integral.

$$\int \frac{1}{x^2 \sqrt{16-x^2}} dx$$

**Question 5.** (5 marks).

(a) Prove the reduction formula

$$\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$$

(b) Use this formula to evaluate

$$\int (\ln x)^3 dx$$

**Question 6.** (2 marks) Evaluate the following integral.

$$\int_{-\pi/3}^{\pi/3} \frac{(x^6 - x^2 + 2) \sin x}{\cos x} dx$$

**Question 7.** (5 marks) Evaluate the following integral.

$$\int_{3\pi/4}^{\pi} \sec^6 x dx$$

**Question 8.** (5 marks) Evaluate the following integral.

$$\int \frac{x^2 + x - 18}{x^3 + 9x} dx$$



**Question 9.** (5 marks) Evaluate the following integral.

$$\int \frac{x^4 + 6x^2 - 20x + 1}{x^2 + 2x + 10} dx$$

**Question 10.** (3 marks). If  $f$  is continuous and

$$\int_1^4 f(x) dx = 24 \quad \text{find} \quad \int_0^1 x^3 f(3x^4 + 1) dx.$$

Clearly show your work.

**Bonus.** (3 marks) Find

$$\int \frac{1}{x^4+1} dx$$

given that

$$\frac{1}{x^4+1} = \frac{\frac{1}{2\sqrt{2}}x + \frac{1}{2}}{x^2 + \sqrt{2}x + 1} + \frac{-\frac{1}{2\sqrt{2}}x + \frac{1}{2}}{x^2 - \sqrt{2}x + 1}$$