

Quiz 7

Question 1. (10 marks) Find the derivative $\frac{dy}{dx}$

(a)

$$\sqrt{xy} = 2x + y^2$$

$$\frac{d}{dx} [\sqrt{xy}] = \frac{d}{dx} [2x] + \frac{d}{dx} [y^2]$$

$$\frac{1}{2} (xy)^{-1/2} \cdot \left(y + x \frac{dy}{dx} \right) = 2 + 2y \frac{dy}{dx}$$

$$\frac{1}{2} x (xy)^{-1/2} \frac{dy}{dx} - 2y \frac{dy}{dx} = 2 - \frac{1}{2} y (xy)^{-1/2}$$

$$\frac{dy}{dx} \left(\frac{1}{2} x (xy)^{-1/2} - 2y \right) = 2 - \frac{1}{2} y (xy)^{-1/2}$$

$$\frac{dy}{dx} = \frac{2 - \frac{1}{2} y (xy)^{-1/2}}{\frac{1}{2} x (xy)^{-1/2} - 2y}$$

(b)

$$\frac{x+y}{x-y} = 3x$$

$$\frac{d}{dx} \left[\frac{x+y}{x-y} \right] = \frac{d}{dx} [3x]$$

$$\frac{\left(1 + \frac{dy}{dx} \right) (x-y) - (x+y) \left(1 - \frac{dy}{dx} \right)}{(x-y)^2} = 3$$

$$\frac{dy}{dx} [x-y \cdot x+y] = 3(x-y)^2 + 2y$$

$$\frac{dy}{dx} = \frac{3(x-y)^2 + 2y}{2x}$$

$$x-y + \frac{dy}{dx} (x-y) - x - y + \frac{dy}{dx} (x+y) = 3(x-y)^2$$

(c)

$$xy^{3/2} = x^2 + y^2$$

$$\frac{d}{dx} [xy^{3/2}] = \frac{d}{dx} [x^2] + \frac{d}{dx} [y^2]$$

$$y^{3/2} + x \left(\frac{3}{2} y^{1/2} \frac{dy}{dx} \right) = 2x + 2y \frac{dy}{dx}$$

$$\frac{dy}{dx} \left(\frac{3}{2} x y^{1/2} - 2y \right) = 2x - y^{3/2}$$

$$\frac{dy}{dx} = \frac{2x - y^{3/2}}{\frac{3}{2} x y^{1/2} - 2y}$$