Dawson College: Calculus I: 201-NYA-05 08 | April 8, 2011

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

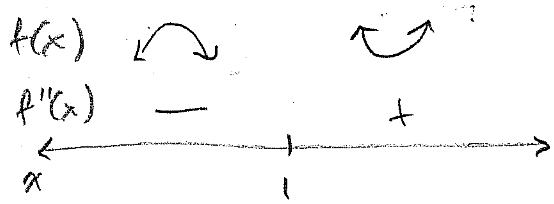
Quiz 8

Question 1.(*5 marks*) Determine where the following function is concave upward and where it is concave downward. Find any inflection points.

$$f(x) = 6x^3 - 18x^2 + 12x - 15 \Rightarrow f'(x) = 18x^2 - 36x + 12$$

$$f''(x) = 36x - 36 = 36(x-1) = 0 \quad \Big| \quad \cancel{f''(x) \text{ D.N.E.}}$$

$$\Rightarrow x = 1$$



TEST POINTS

$x=0 \quad f''(0) = -36 < 0$

$x=2 \quad f''(2) = 36 > 0$

$f(x)$ IS CONCAVE DOWNWARD ON $(-\infty, 1)$ AND CONCAVE UPWARD ON $(1, \infty)$

$f(1) = 6 - 18 + 12 - 15 = -15 \quad \therefore (1, -15)$ IS AN INFLECTION POINT.

Question 2.(*5 marks*) Use the second derivative test to find any relative ~~extrema~~ extrema of:

$$f(x) = \frac{x}{1+x^2} \qquad f'(x) = \frac{(1)(1+x^2) - x(2x)}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$f'(x) = 0$ | $f'(x)$ D.N.E

$1 - x^2 = 0$ | $f'(x)$ ALWAYS

$x = \pm 1$ | EXIST SINCE $(1+x^2)^2 \neq 0$

$$f''(x) = \frac{(-2x)(1+x^2)^2 - (1-x^2)[2(1+x^2)\cdot(2x)]}{(1+x^2)^4}$$

$$f''(1) = \frac{(-2)(2)^2 - (0)}{(2)^4} = -\frac{1}{2} < 0 \Rightarrow f(1) = \frac{1}{2} \text{ IS A REL. MAX}$$

$$f''(-1) = \frac{2(2)^2 - (0)}{2^4} = \frac{1}{2} > 0 \Rightarrow f(-1) = -\frac{1}{2} \text{ IS A REL MIN.}$$