Dawson College: Calculus I: 201-NYA-05 08 February 24, 2011

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Test 1

The time for this test is 1 hour and 45 minutes. Remember to use correct notation for full marks.

Question 1. *(8 marks)* Simplify the following expressions. Write your final answer as a single fraction with positive exponents only.

(a) $\left(\frac{a^2b^{1/2}}{c}\right)\cdot\left(\frac{c^8b^4}{16a^2b^{-2}}\right)^{-1/4} = \frac{a^2b^{1/2}}{c}\cdot\frac{(c^8)^{-1/4}(b^4)^{-1/4}}{(16)^{-1/4}(a^2)^{-1/4}(b^{-2})^{-1/4}}$

$$= \frac{a^2b^{1/2}}{c}\cdot\frac{c^{-2}b^{-1}}{16^{-1/4}a^{-1/2}b^{1/2}} = \frac{a^2b^{1/2}}{c}\cdot\frac{16^{1/4}a^{1/2}}{c^2b^1b^{1/2}}$$

$$= \frac{2a^2a^{1/2}b^{1/2}}{cc^2b^1b^{1/2}} = \frac{2a^{5/2}b^{1/2}}{c^3b^{3/2}} = \frac{2a^{5/2}}{c^3b}$$

(b) $\frac{(5x-3)^{1/2}-(5x-3)^{-1/2}(x-6)}{4x+3} = \frac{(5x-3)^{1/2}-\frac{x-6}{(5x-3)^{1/2}}}{4x+3}$

$$= \frac{\frac{(5x-3)^{1/2}\cdot(5x-3)^{1/2}}{(5x-3)^{1/2}}-\frac{x-6}{(5x-3)^{1/2}}}{4x+3} = \frac{\frac{(5x-3)-(x-6)}{(5x-3)^{1/2}}}{4x+3}$$

$$= \frac{\frac{4x+3}{(5x-3)^{1/2}}}{4x+3} = \frac{4x+3}{(5x-3)^{1/2}}\cdot\frac{1}{4x+3} = \frac{1}{(5x-3)^{1/2}}$$

Question 2. Find the following limits if they exist.

(a) *(2 marks)*

$$\lim_{x\to 2}\frac{x\sqrt{6x-12}}{x+\sqrt{3x+3}} = \frac{2\sqrt{6(2)-12}}{2+\sqrt{3(2)+3}} = \frac{2\cdot 0}{2\sqrt{9}} = \frac{0}{6} = 0$$

(b) *(3 marks)*

$$\lim_{x\to -5}\frac{x^2+9x+20}{x^2+5x} \underset{\text{I.F. } \frac{0}{0}}{=} \lim_{x\to -5}\frac{(x+4)\cancel{(x+5)}}{x\cancel{(x+5)}}$$

$$= \lim_{x\to -5}\frac{x+4}{x} = \frac{-5+4}{-5} = \frac{1}{5}$$

(c) *(3 marks)*

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2} \underset{\text{I.F. } \frac{0}{0}}{=} \lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$= \lim_{x\to 4}\frac{(x-4)(\sqrt{x}+2)}{x-2\sqrt{x}+2\sqrt{x}-4} = \lim_{x\to 4}\frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}}$$

$$= \lim_{x\to 4}(\sqrt{x}+2) = \sqrt{4}+2 = 4.$$

(d) *(3 marks)*

$$\lim_{x\to\infty} \frac{4x^5 - 3x^2 + 2}{-x^5 + 6x^3 + 9x} = \lim_{x\to\infty} \frac{\frac{4x^5}{x^5} - \frac{3x^2}{x^5} + \frac{2}{x^5}}{-\frac{x^5}{x^5} - \frac{6x^3}{x^5} + \frac{9x}{x^5}}$$

$$= \lim_{x\to\infty} \frac{4 - 3/x^3 + 2/x^5}{-1 - 6/x^2 + 9/x^4} = \frac{4 - 0 + 0}{-1 - 0 + 0} = -4$$

(e) *(3 marks)*

$$\lim_{x\to-\infty} \frac{4x^4 + 2x - 1}{2x^2 + x} = \lim_{x\to-\infty} \frac{\frac{4x^4}{x^2} + \frac{2x}{x^2} - 1/x^2}{\frac{2x^2}{x^2} + \frac{x}{x^2}}$$

$$= \lim_{x\to-\infty} \frac{\overset{\nearrow +\infty}{4x^2} + \overset{\nearrow 0}{2/x} - \overset{\nearrow 0}{1/x^2}}{\underset{\searrow 2}{2} + \underset{\searrow 0}{1/x}} = \infty$$

THE LIMIT D.N.E.

Question 3. *(5 marks)* Where are the following function continuous. (Explain your reasoning using the conditions for continuity)

$$f(x) = \begin{cases} 3x^2 - 2 & \text{if } x < 2 \\ 11 & \text{if } x = 2 \\ 3x + 4 & \text{if } x > 2 \end{cases}$$

• f IS CONTINUOUSE EVERYWHERE EXCEPT POSSIBLY AT $x=2$ SINCE IT IS A POLYNOMIAL ON THESE INTERVALS.

AT $\underline{x=2}$ WE WILL CHECK THE THREE CONDITIONS OF CONTINUITY

1) $f(2) = 11$

2) $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} (3x^2 - 2) = 3(2)^2 - 2 = 10$

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} (3x+4) = 3(2)+4 = 10$

$\therefore \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x)$ SO $\lim\limits_{x \to 2} f(x)$ EXISTS (AND $= 10$)

3) $f(2) = 11 \neq 10 = \lim\limits_{x \to 2} f(x)$ FAILS CONDITION 3

$\therefore$ f IS NOT CONTINUOUS AT $x = 2$

$\therefore$ f IS CONTINUOUS ON

$(-\infty, 2) \cup (2, \infty)$

Question 4. *(5 marks)* Use the limit definition of the derivative to find $f'(x)$ given $f(x) = 2x^2 - 3x + 1$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[2(x+h)^2 - 3(x+h) + 1] - [2x^2 - 3x + 1]}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}}$$

$$= \lim_{h \to 0} (4x + 2h - 3)$$

$$= 4x + 2(0) - 3$$

$$= 4x - 3$$

Question 5. Find the derivatives of the following functions. Express your final answer with positive exponents.

(a) *(3 marks)*

$$f(x) = 5x^{4/3} - \frac{2}{3}x^{3/2} + \sqrt{x}$$

$$f'(x) = 5\left(\frac{4}{3}x^{1/3}\right) - \frac{2}{3}\left(\frac{3}{2}x^{1/2}\right) + \frac{1}{2}x^{-1/2}$$

$$= \frac{20}{3}x^{1/3} - x^{1/2} + \frac{1}{2x^{1/2}}$$

(b) *(3 marks)*

$$g(x) = (x^{1/2} + 3x^2)\left(4x^3 - \frac{1}{x^3}\right)$$

$$g'(x) = \left(\frac{1}{2}x^{-1/2} + 6x\right)\left(4x^3 - \frac{1}{x^3}\right) + (x^{1/2} + 3x^2)(12x^2 + 3x^{-4})$$

$$= \left(\frac{1}{2x^{1/2}} + 6x\right)\left(4x^3 - \frac{1}{x^3}\right) + (x^{1/2} + 3x^2)\left(12x^2 + \frac{3}{x^4}\right)$$

(c) *(3 marks)*

$$h(x) = \frac{x^2 + x - 1}{3x^3 + 2x}$$

$$h'(x) = \frac{(2x+1)(3x^3+2x) - (x^2+x-1)(9x^2+2)}{(3x^3+2x)^2}$$

Question 6. *(4 marks)* Find the points on the graph of $f(x) = x^3 - 3x + 5$ where the tangent line is horizontal.

$$f'(x) = 3x^2 - 3 = 0$$

$$3(x^2-1) = 0$$

$$3(x+1)(x-1) = 0$$

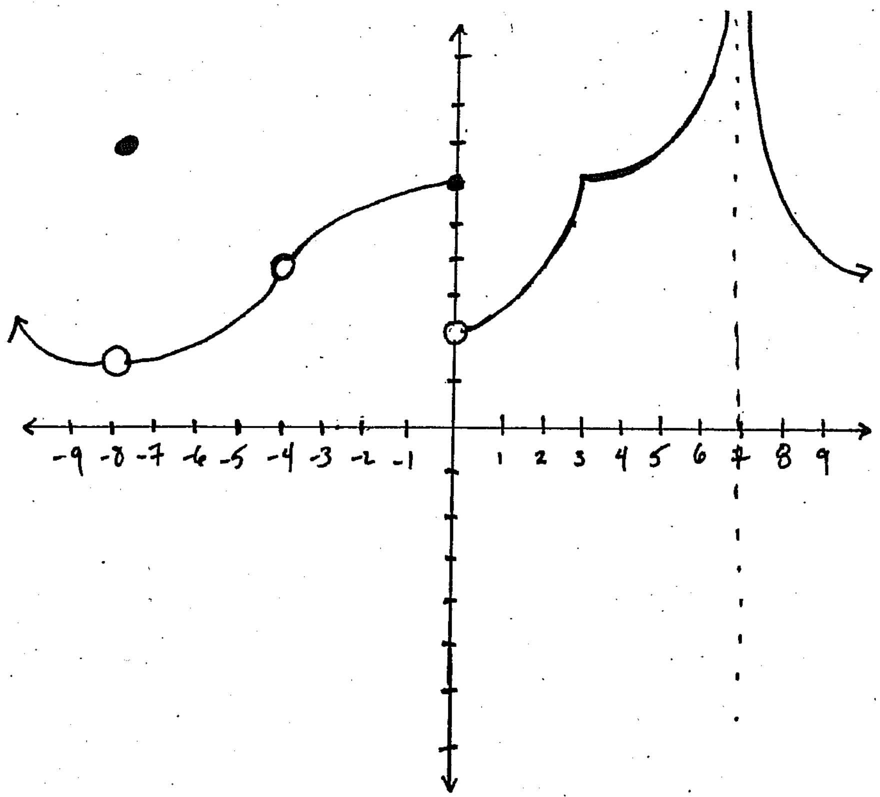
$$x = -1, 1$$

$$f(-1) = (-1)^3 - 3(-1) + 5 = 7$$

$$f(1) = (1)^3 - 3(1) + 5 = 3$$

$\therefore$ THE GRAPH HAS A HORIZONTAL TANGENT LINE AT $(-1, 7)$ AND $(1, 3)$.

Question 7. *(5 marks)*
(a)) State where $f(x)$ is is discontinuous and why (state which condition of continuity it fails).
(b)) State where $f(x)$ is not differentiable and why.



a) f IS NOT CONTINUOUS AT:

$x=-8$, FAILS CONDITION 3
$x=-4$, FAILS CONDITION 1
$x=0$, FAILS CONDITION 2
$x=7$, FAILS CONDITION 1

b) f IS NOT DIFFERENTIABLE AT

$x=-8,-4,0$ AND 7 BECAUSE IT IS NOT CONTINUOUS THERE.

f IS NOT DIFFERENTIABLE AT $x=3$ BECAUSE IT HAS A CORNER THERE (CHANGES DIRECTION DRASTICALLY)