Dawson College: **Calculus I**: 201-NYA-05 08

April 1, 2011

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Test 2

The time for this test is 1 hour and 45 minutes. Remember to use correct notation for full marks.

Question 1. *(6 marks)* Find the following derivatives. (Do not simplify)

(a) $f(x) = \dfrac{\sqrt{2x+1}}{(x^2-1)^3}$.

$$f'(x) = \frac{\frac{1}{2}(2x+1)^{-1/2}(2)(x^2-1)^3 - (2x+1)^{1/2}\cdot 3(x^2-1)^2\cdot(2x)}{(x^2-1)^6}$$

(b) $g(t) = (1+t^2)^5(1-2t^2)^8$

$$g'(t) = 5(1+t^2)^4\cdot(2t)\cdot(1-2t^2)^8 + (1+t^2)^5\cdot 8(1-2t^2)^7(-4t)$$

Question 2. Given $f(1) = 2$, $g(1) = 1$, $f'(1) = -1$, $g'(1) = 3$ and

$$h(x) = \frac{x + f(x)}{x \cdot g(x)}$$

find $h'(1)$.

$$h'(x) = \frac{\frac{d}{dx}[x + f(x)] \cdot (x \cdot g(x)) - [x + f(x)] \frac{d}{dx}[x \cdot g(x)]}{[x \cdot g(x)]^2}$$

$$= \frac{[1 + f'(x)]\, x \cdot g(x) - (x + f(x))[1 \cdot g(x) + x g'(x)]}{[x \cdot g(x)]^2}$$

$$\therefore h'(1) = \frac{[1 + f'(1)] \cdot 1 \cdot g(1) - (1 + f(1))[g(1) + 1 \cdot g'(1)]}{[1 \cdot g(1)]^2}$$

$$= \frac{(1 + (-1)) \cdot 1 \cdot 1 - (1 + 2)[1 + 1 \cdot 3]}{[1 \cdot 1]^2}$$

$$= \frac{0 - 12}{1}$$

$$= -12$$

Question 3. *(5 marks)* The demand function for the Acrosonic speaker system is

$$p = -0.04x + 800$$

where p is the price per unit (in dollars) and x denotes the quantity demanded. The cost of producing x units is given by

$$C(x) = 200x + 300\,000.$$

(a) Find the ~~revenue~~ profit function ~~R~~ P and the marginal revenue function ~~R′~~ P′.
(b) Compute ~~R′~~ P′(5000) and interpret the result.

a) $R(x) = px = (-0.04x + 800)x = -0.04x^2 + 800x$

$P(x) = R(x) - C(x) = (-0.04x^2 + 800x) - (200x + 300\,000)$

$= -0.04x^2 + 600x - 300\,000$

$P'(x) = -0.08x + 600$

b) $P'(5000) = -0.08(5000) + 600 = 200$

$\therefore$ THE ~~[illegible]~~ PROFIT REALIZED FROM THE SALE OF THE 5001ST UNIT IS APPROXIMATELY \$200

Question 4. *(5 marks)* The quantity demanded each week x (in units of 100) of the Compustar webcamera is

$$x = \sqrt{400 - 5p}$$

(a) Is demand elastic or inelastic when p=60? What should Compustar do to the price to increase revenue?
(b) At what price is demand unitary?

$$f(p) = \sqrt{400-5p}$$

$$f'(p) = \frac{1}{2}(400-5p)^{-1/2}(-5) = \frac{-5}{2(400-5p)^{1/2}}$$

$$\therefore E(p) = \frac{-p\,f'(p)}{f(p)} = \frac{-p\left(\frac{-5}{2(400-5p)^{1/2}}\right)}{\sqrt{400-5p}}$$

$$= \frac{5p}{2(400-5p)} \Rightarrow E(60) = \frac{5(60)}{2(400-5(60))} = \frac{3}{2} > 1$$

$\therefore$ DEMAND IS ELASTIC WHEN $p = 60$.

COMPUSTAR SHOULD DECREASE THE PRICE TO INCREASE REVENUE.

b) $$\frac{5p}{2(400-5p)} = 1 \Rightarrow 5p = 2(400-5p)$$

$$5p = 800 - 10p$$

$$15p = 800$$

$$p = \frac{800}{15} = 53.33$$

DEMAND IS UNITARY AT \$53.33

Question 5. *(5 marks)* Find the third derivative of $f(x) = \sqrt{x^2 - 3}$

$$f'(x) = \frac{1}{2}(x^2-3)^{-1/2} \cdot (2x) \quad = \quad x(x^2-3)^{-1/2}$$

$$f''(x) = 1 \cdot (x^2-3)^{-1/2} + x\left[-\frac{1}{2}(x^2-3)^{-3/2} \cdot (2x)\right]$$

$$= (x^2-3)^{-1/2} \quad - x^2(x^2-3)^{-3/2}$$

$$f'''(x) = -\frac{1}{2}(x^2-3)^{-3/2}(2x) - 2x(x^2-3)^{-3/2}$$

$$- x^2\left[-\frac{3}{2}(x^2-3)^{-5/2}(2x)\right]$$

$$= -3x(x^2-3)^{-3/2} + 3x^3(x^2-3)^{-5/2}$$

Question 6. *(6 marks)*
(a) Find $\frac{dy}{dx}$ given $x^2y^3 - 2y^2 + (x+y)^3 = 5$
(b) Find the tangent line to the graph of the above equation at $(1,1)$

a) $\frac{d}{dx}[x^2y^3] - \frac{d}{dx}[2y^2] + \frac{d}{dx}[(x+y)^3] = \frac{d}{dx}[5]$

$2xy^3 + x^2 \cdot 3y^2\frac{dy}{dx} - 4y\frac{dy}{dx} + 3(x+y)^2 \cdot (1 + \frac{dy}{dx}) = 0$

$3x^2y^2\frac{dy}{dx} - 4y\frac{dy}{dx} + 3(x+y)^2\frac{dy}{dx}$

$= -2xy^3 - 3(x+y)^2$

$\frac{dy}{dx}[3x^2y^2 - 4y + 3(x+y)^2] = -2xy^3 - 3(x+y)^2$

$$\boxed{\frac{dy}{dx} = \frac{-2xy^3 - 3(x+y)^2}{3x^2y^2 - 4y + 3(x+y)^2}}$$

b) $a = \frac{dy}{dx}\Big|_{(1,1)} = \frac{-2(1)(1)^3 - 3(1+1)^2}{3(1)^2(1)^2 - 4(1) + 3(1+1)^2}$

$= \frac{-2 - 12}{3 - 4 + 12} = -\frac{14}{11}$

$y = ax + b$

$1 = -\frac{14}{11}(1) + b$

$1 + \frac{14}{11} = b$

$\frac{25}{11} = b$

$\therefore \boxed{y = -\frac{14}{11}x + \frac{25}{11}}$

Question 7. *(5 marks)* Suppose the demand equation for a certain product is

$$180x^2 + 10p^2 = 4100$$

where x represents the number of units (in the thousands) demanded each week when the price is \$$p$. How much is the quantity demanded increasing when the price is \$12 per unit and the unit price decreasing at a rate of \$0.21 per unit per week? (You can use decimals, round your final answer to the nearest unit.)

$p = 12 \quad \frac{dp}{dt} = -0.21$

$180x^2 + 10(12)^2 = 4100 \Rightarrow 180x^2 = 4100 - 10(12)^2$

$x^2 = 14.\overline{7}$

$x = 3.8442$

$\frac{d}{dt}[180x^2] + \frac{d}{dt}[10p^2] = \frac{d}{dt}[4100]$

$360x\frac{dx}{dt} + 20p\frac{dp}{dt} = 0$

$360(3.8442)\frac{dx}{dt} + 20(12)(-0.21) = 0$

$1383.908\frac{dx}{dt} = 50.4$

$\frac{dx}{dt} = 0.0364$

$\therefore$ QUANTITY DEMANDED IS INCREASING BY $(0.036)(1000) = 36$ UNITS PER WEEK AT THE TIME UNDER CONSIDERATION.

Question 8. *(5 marks)* Find the intervals where

$$f(x) = \frac{x^4}{4} - 2x^3 + \frac{9}{2}x^2 + 10$$

is increasing and where it is decreasing. Find the relative extrema.

$$f'(x) = x^3 - 6x^2 + 9x = 0$$

$$x(x^2 - 6x + 9) = 0$$

$$x(x-3)(x-3) = 0$$

$$\therefore\ x = 0, 3$$

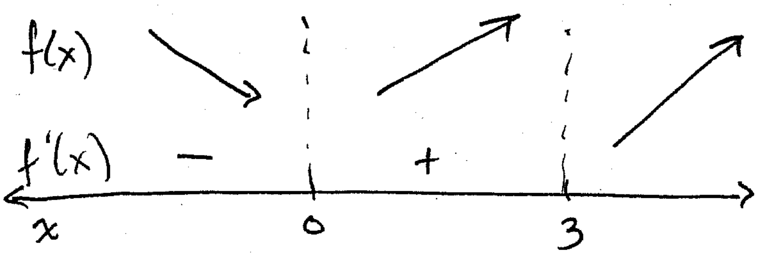
($f'(x)$ ALWAYS EXISTS SO THESE ARE THE ONLY C.N.)

TEST POINTS

$x = -1 : f'(-1) = -16 < 0$

$x = 1 : f'(1) = 4 > 0$

$x = 4 : f'(4) = 4 > 0$



$\therefore$ f IS DECREASING ON $(-\infty, 0)$ AND

INCREASING ON $(0, 3)$ AND $(3, \infty)$

$f(0) = 10$ IS A RELATIVE MINIMUM.