Dawson College: Calculus I: 201-NYA-05 08 April 29, 2011

Test 3

The time for this test is 1 hour and 45 minutes. Remember to use correct notation for full marks.

Question 1. *(4 marks)* Determine the relative extrema of

$$f(x) = \frac{2}{3}x^3 + x^2 - 12x + 5$$

Question 2. *(8 marks)* Find the domain, intercepts and asymptotes of the following functions:

$$f(x) = \frac{x^2 + 2x - 8}{x^2 - 4x - 5}$$

(b) $f(x) = \dfrac{x^2 - 1}{x(x+2)(x-3)}$

Question 3. *(4 marks.)* The function

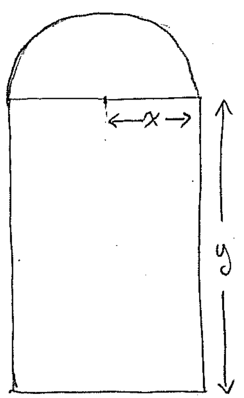
$$f(x) = \frac{4 - 4x}{x^2}$$

has the following properties:
Domain: $(-\infty, 0) \cup (0, \infty)$. **x-intercept,** : $(-1, 0)$.
Horizontal asymptote: $y = 0$. **Vertical asymptote:** $x = 0$.
f **is** decreasing **on:** $(-2, 0)$. f **is** increasing **on:** $(-\infty, -2)$ and $(0, \infty)$
Relative minimum: none. **Relative maximum:** $(-2, 1)$
Concave downward on: $(-3, 0)$ and $(0, \infty)$.
Concave upward on: $(-\infty, -3)$.
Inflection Points: $(-3, \frac{8}{9})$. Sketch the graph of $f(x)$.

Question 4. *(10 marks.)* Sketch the graph of $f(x) = x^4 - 2x^2$.

Question 5. *(4 marks.)* Find the absolute maximum and absolute minimum of
$f(x) = 2x^3 - 3x^2 - 12x$ on $[-1, 2]$.

Question 6. *(5 marks.)* A Norman window has the shape of a rectangle surmounted by a semicircle. If a norman window is to have a perimeter of 28ft, what should it's dimentions be in order to allow the maximum amount of light throught the window?



Question 7. *(5 marks.)* A closed rectangular container with a square base is to have a volume of 2250in^3. The material for the top and bottom of the container will cost \$2 per in^2, and the material for the sides will cost \$3 per in^2. Find the dimensions of the container of least cost.

Question 8. *(2 marks.)* Express the $\ln x + \frac{1}{2}\ln(x^2+1) - 2\ln(x+2)$ as a single logarithm.

Question 9. *(5 marks.)* Solve for x.

(a) $\log_2 5 + \log_2 x = 1$

(b) $9^x = \dfrac{1}{27^{2x+2}}$

$f'(x) = 2x^2 + 2x - 12 = 0$

$2(x^2 + x - 6) = 0$

$(x+3)(x-2) = 0$

$\therefore \; x = -3, 2$

$f''(x) = 4x + 2$

$f''(-3) = 4(-3) + 2 = -10 < 0$

$\therefore \; f(-3) = \frac{2}{3}(-3)^3 + (-3)^2 - 12(-3) + 5 = 32$

IS A RELATIVE MAXIMUM.

$f''(2) = 4(2) + 2 = 10 > 0$

$\therefore \; f(2) = \frac{2}{3}(2)^3 + (2)^2 - 12(2) + 5 = -29/3$

IS A RELATIVE MINIMUM

(ACCORDING TO THE SECOND DERIVATIVE TEST.)

2a) $x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = -1, 5$

$\therefore$ DOMAIN OF f: $(-\infty, -1) \cup (-1, 5) \cup (5, \infty)$

x-int: $y=0$

$$0 = \frac{x^2+2x-8}{x^2-4x-5} = \frac{(x+4)(x-2)}{(x-5)(x+1)}$$

$\Rightarrow x = -4, 2$

$\therefore (-4, 0), \quad (2, 0)$

y-int

$$y = \frac{0+0-8}{0-0-5} = \frac{8}{5} \qquad \therefore \left(0, \tfrac{8}{5}\right)$$

$x=-1$ AND $x=5$ ARE VERTICAL ASYMPTOTES

(DENOMINATOR $=0$, NUMERATOR $\neq 0$)

$$\lim_{x\to\infty} \frac{x^2+2x-8}{x^2-4x-5} = \lim_{x\to\infty} \frac{1+2/x-8/x^2}{1-4/x-5/x^2} = \frac{1+0-0}{1-0-0} = 1$$

$$\lim_{x\to-\infty} \frac{x^2+2x-8}{x^2-4x-5} = \lim_{x\to-\infty} \frac{1+2/x-8/x^2}{1-4/x-5/x^2} = 1$$

$\therefore y=1$ IS THE HORIZONTAL ASYMPTOTE.

b) DOMAIN: $(-\infty,-2)\cup(-2,0)\cup(0,3)\cup(3,\infty)$

x-int:

$$0 = \frac{x^2-1}{x(x+2)(x-3)} \Rightarrow \quad 0 = x^2-1 \Rightarrow x = \pm 1$$

$\therefore$ $(1,0)$, $(-1,0)$

y-int: NO y-int SINCE $x=0$ IS NOT IN THE DOMAIN.

$x=0$, $x=-2$ AND $x=3$ ARE VERTICAL ASYMPTOTES

DENOMINATOR $=0$, NUMERATOR $\neq 0$)

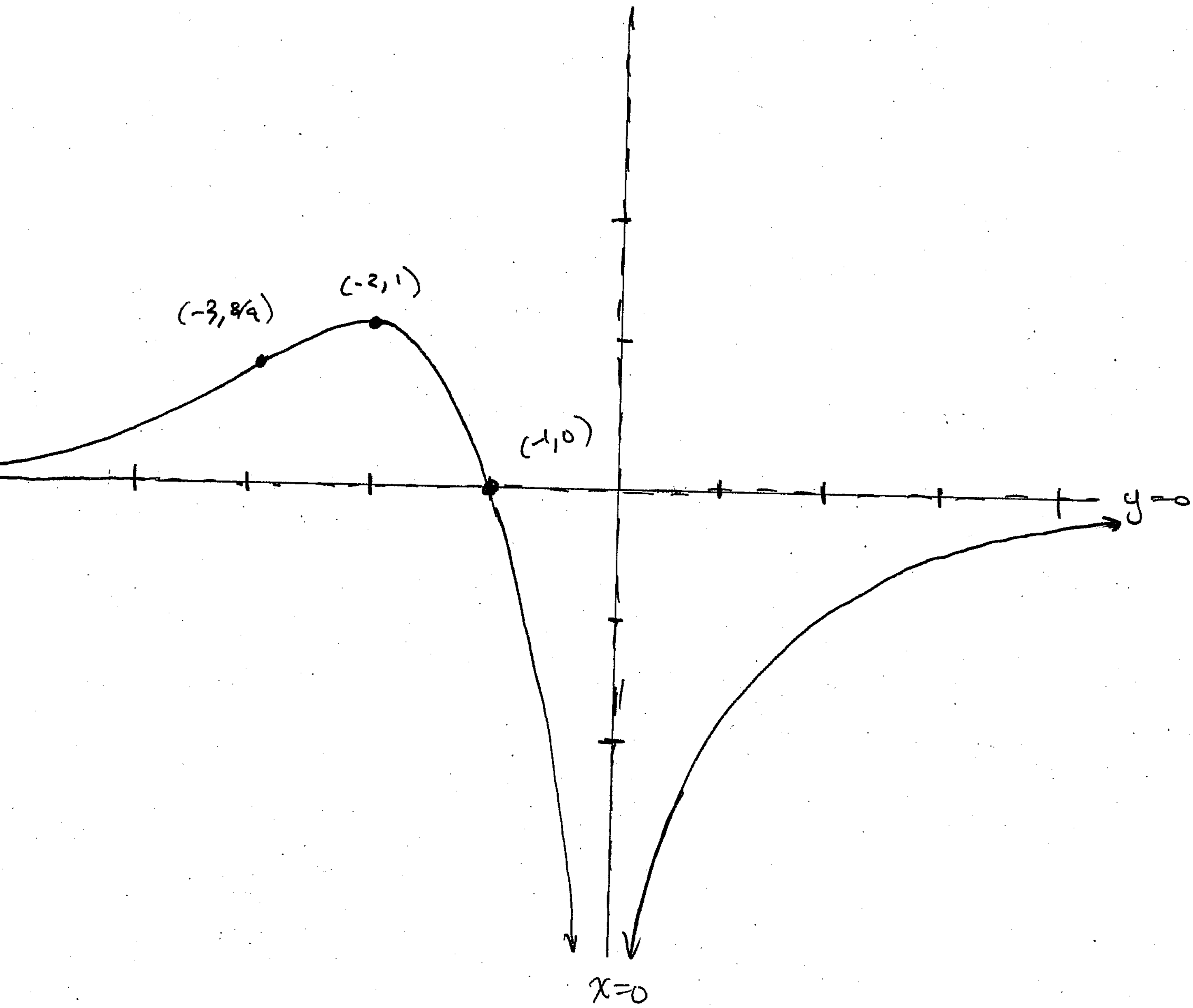
$$\lim_{x\to\infty} \frac{x^2-1}{x(x+2)(x-3)} = \lim_{x\to\infty} \frac{x^2-1}{x^3-x^2-6x} = \lim_{x\to\infty} \frac{1/x - 1/x^3}{1-1/x-6/x^2}$$

$$= \frac{0-0}{1-0-0} = 0$$

$$\lim_{x\to-\infty} \frac{1/x - 1/x^3}{1-1/x-6/x^3} = 0$$

$\therefore$ $y=0$ IS THE HORIZONTAL ASYMPTOTE.

3)



4)

1) DOMAIN: $\mathbb{R}$ (POLYNOMIAL)

2) y-int: $y = 0^4 - 2(0)^2 = 0 \quad \therefore (0,0)$

x-int: $0 = x^4 - 2x^2 = x^2(x^2-2)$

$\Rightarrow x = 0, \pm\sqrt{2}$

$\therefore (0,0), (-\sqrt{2}, 0), (\sqrt{2}, 0)$

3) NO V.A. (POLYNOMIAL)

4) $\lim\limits_{x\to\infty} f(x) = \infty$

$\lim\limits_{x\to-\infty} f(x) = \infty$

NO H.A.

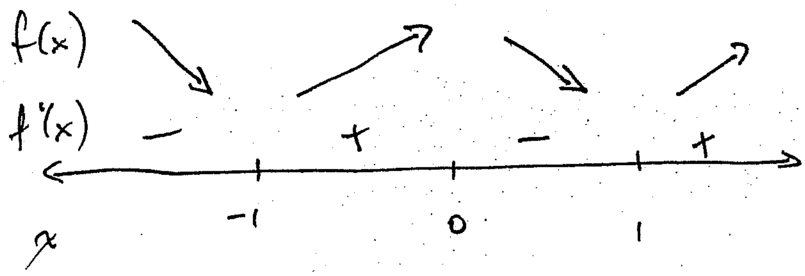
5) $f'(x) = 4x^3 - 4x = 0$

$4x(x^2-1) = 0$

$4x(x+1)(x-1) = 0$

$x = 0, -1, 1$

~~$f'(x)$ D.N.E.~~



TEST POINTS.

$f'(-2) = -24$

$f'(-1/2) = 3/2$

$f'(1/2) = -3/2$

$f'(2) = 24$

$\therefore$ f IS DECREASING ON $(-\infty, -1)$ AND $(0, 1)$ AND

f IS INCREASING ON $(-1, 0)$ AND $(1, \infty)$

f HAS RELATIVE MAXIMUM

$f(0) = 0$

AND RELATIVE MINIMA

$f(-1) = -1$

$f(1) = -1$

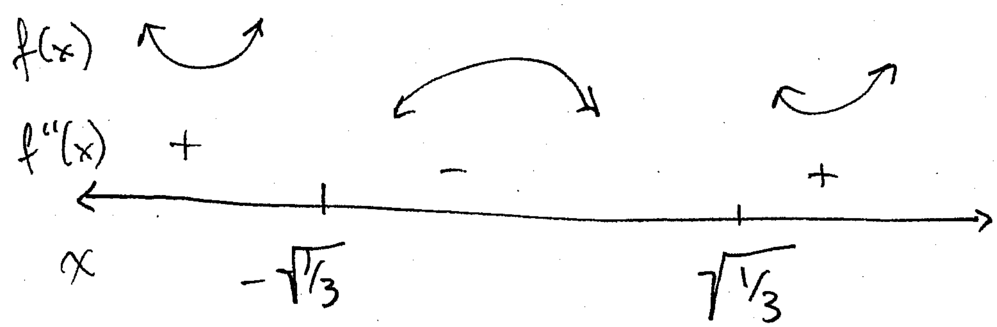
e) $f''(x) = 12x^2 - 4 = 0$

$4(3x^2 - 1) = 0$

$3x^2 - 1 = 0$

$x = \pm\sqrt{1/3}$

| ~~$f''(x)$ D.N.E.~~



TEST POINTS

$f''(-1) = 8$

$f''(0) = -4$

$f''(1) = 8$

$\therefore$ f IS CONCAVE UPWARD ON $(-\infty, -\sqrt{1/3})$ AND $(\sqrt{1/3}, \infty)$ AND f IS CONCAVE DOWNWARD ON $(-\sqrt{1/3}, \sqrt{1/3})$

$$f(\sqrt{1/3}) = (\sqrt{1/3})^4 - 2(\sqrt{1/3})^2 = \frac{1}{9} - \frac{2}{3} = -5/9$$

$$f(-\sqrt{1/3}) = -5/9$$

$\therefore$ $(-\sqrt{1/3}, -5/9)$ AND $(\sqrt{1/3}, 5/9)$ ARE INFLECTION POINTS.

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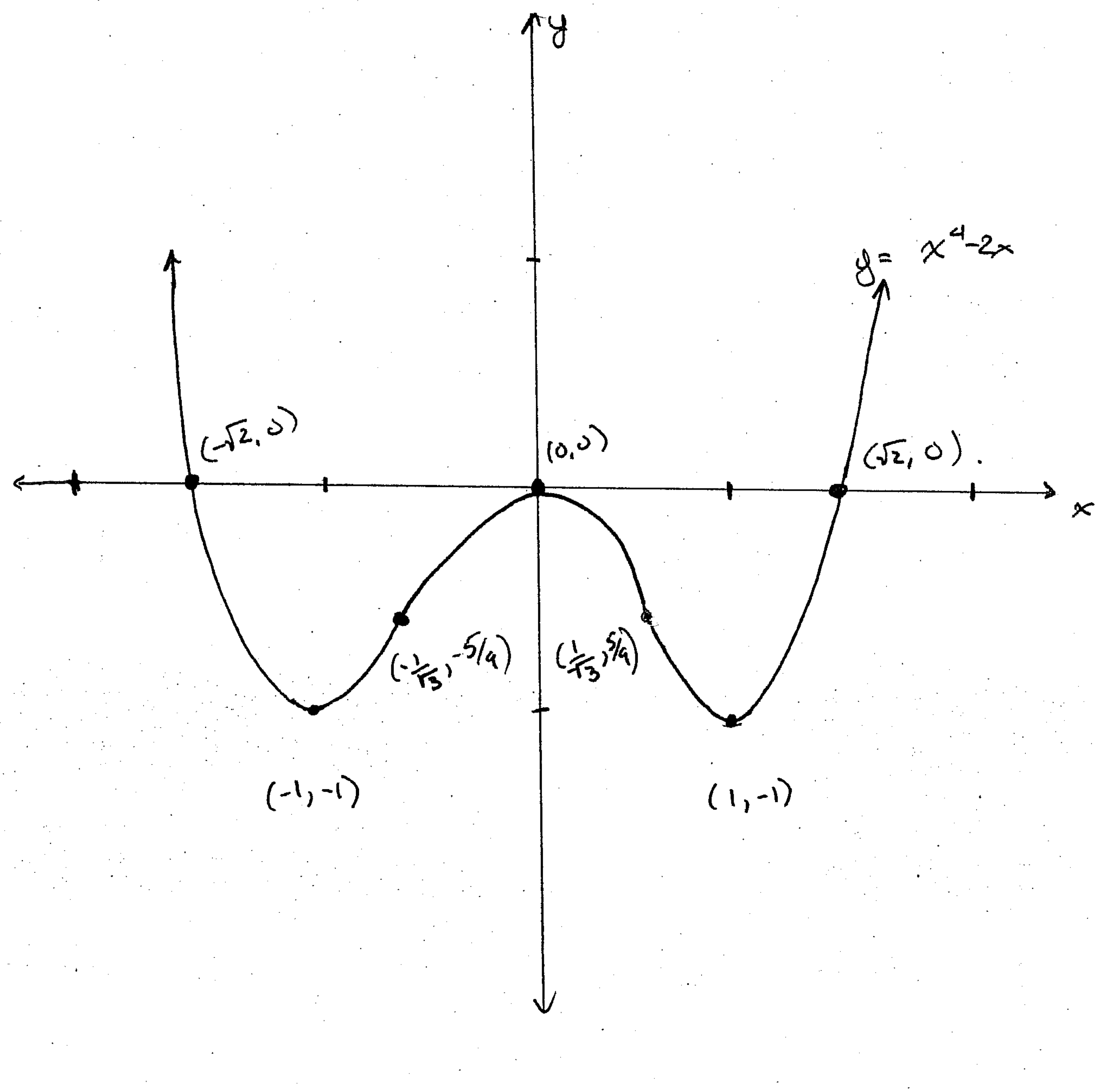


5. $f'(x) = 6x^2 - 6x - 12$

$= 6(x^2 - x - 2)$

$= 6(x-2)(x+1) = 0$

$\Rightarrow \quad x = -1, 2$

C. N.

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) = 7$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) = -20$

END POINTS

$f(-1) = 7$

$f(2) = -20$

$\therefore$ $(-1, 7)$ IS THE ABSOLUTE MAX

$(2, -20)$ IS THE ABSOLUTE MIN.

6.

2) $A = 2xy + \frac{1}{2}\pi x^2$

3) $2x + 2y + \pi x = 28$

$\Rightarrow \quad y = 14 - x - \frac{\pi}{2}x$

$\therefore \quad A = 2x\left(14 - x - \frac{\pi}{2}x\right) + \frac{1}{2}\pi x^2$

$= 28x - 2x^2 - \pi x^2 + \frac{1}{2}\pi x^2$

$= 28x - 2x^2 - \frac{1}{2}\pi x^2$

RESTRICTIONS:

$x \geq 0$

$2x \leq 28$

$x \leq 14$

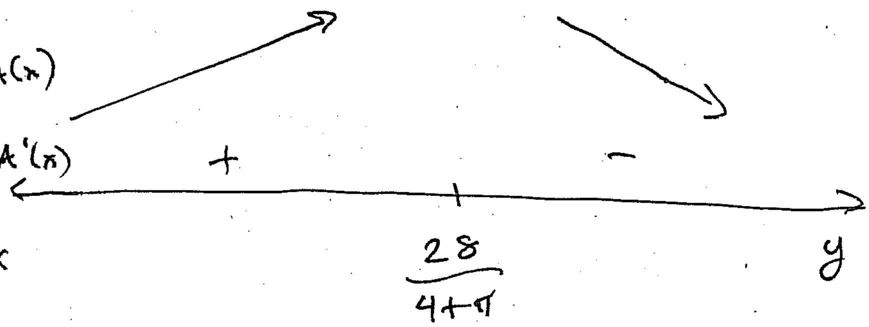
(ACTUALLY SMALLER BUT THIS WILL WORK)

4. $A'(x) = 28 - 4x - \pi x = 0$

$$28 = 4x + \pi x$$

$$28 = (4+\pi)x$$

$$\Rightarrow \quad x = \frac{28}{4+\pi} \approx 3.92$$



$A'(0) = 28$

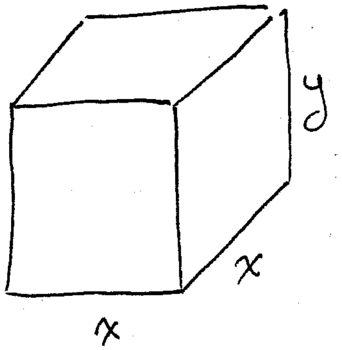
$A'(10) = 28 - 4(10) - \pi(10) = -43.4$

$\therefore \quad x = \frac{28}{4+\pi}$ YIELDS MAXIMUM AREA

$\therefore$ THE DIMENSIONS ARE

$$x = \frac{28}{4+\pi} \approx 3.92\text{ft}, \quad y = 14 - \frac{28}{4+\pi} - \frac{\pi}{2}\left(\frac{28}{4+\pi}\right) \approx 3.92\text{ ft}$$

7)



2. $C = 2(x^2)(2) + 4(xy)(3)$

$= 4x^2 + 12xy$

3. $V = x^2 y = 2250$

$\Rightarrow y = \dfrac{2250}{x^2}$

$\therefore C(x) = 4x^2 + 12x\left(\dfrac{2250}{x^2}\right)$

$= 4x^2 + \dfrac{27\,000}{x}$

RESTRICTIONS:

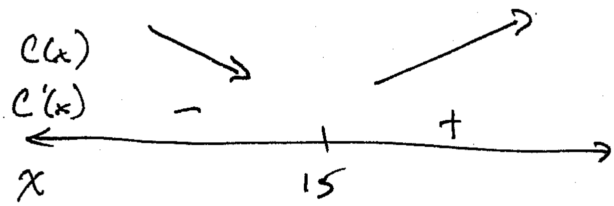
$x > 0$

4. $C'(x) = 8x - \dfrac{27\,000}{x^2} = 0$

$\Rightarrow 8x^3 = 27\,000$

$x^3 = 3375$

$x = 15$



$C'(1) = 8 - 27000 < 0$

$C'(16) = 8(16) - \dfrac{27\,000}{(16)^2} > 0$

$\therefore$ $x = 15$ YIELDS ABS. MIN.

$\therefore$ $x = 15$ in AND $y = \dfrac{2250}{(15)^2} = 10$ in

8. $\ln x + \frac{1}{2}\ln(x^2+1) - 2\ln(x+2)$

$= \ln x + \ln(x^2+1)^{1/2} - \ln(x+2)^2$

$= \ln \dfrac{x\cdot(x^2+1)^{1/2}}{(x+2)^2}$

9. a) $\log_2 5 + \log_2 x = 1$

$\Rightarrow \quad \log_2 5\cdot x = \log_2 2$

$\Rightarrow \quad 5x = 2$

$\Rightarrow \quad x = 2/5$

b) $9^x = \dfrac{1}{(27)^{2x+2}}$

$\Rightarrow \quad (3^2)^x = \dfrac{1}{(3^3)^{2x+2}}$

$\Rightarrow \quad 3^{2x} = \dfrac{1}{3^{6x+6}}$

$\Rightarrow \quad 3^{2x} = 3^{-(6x+6)}$

$\Rightarrow \quad 2x = -6x - 6$

$8x = -6$

$x = -3/4$