1) FIND THE DERIVATIVES OF THE FOLLOWING FUNCTIONS!

a) $f(x) = 5x^2 + 2x^{-2} + \frac{1}{x}$

$g(t) = 2t - 4\sqrt{t} + \frac{3}{\sqrt[3]{t}}$

c) $f(x) = (x^3 + 2x)\left(5 - \frac{1}{x^2}\right)$

d) $f(x) = \frac{\sqrt{x} + 1}{x^2 + 2}$

e) $f(x) = \frac{2}{3}x^3 - (x^2 + 1)(2x^2 - 3x + 1)$

f) $g(x) = \frac{x}{x^2 + 1} - \frac{x - 1}{x^2 - 1}$

g) $f(x) = (-x^2 + 6x)^{54}$

h) $h(t) = \frac{t^2 - 2t}{(t+1)(t^2+1)}$

$f(x) = \frac{\sqrt{3x - 1}}{x^2 + 1}$

j) $f(x) = (x^2 + 1)(x^2 - 3)(5x^2 + x)$

k) $f(x) = \sqrt{(x+3)(x^2 - 1)}$

2) FIND THE EQUATION OF THE TANGENT LINE TO $f(x) = \sqrt{2x+5}$ AT $(2, 3)$

3) FIND $h'(2)$ GIVEN

$h(x) = \frac{[f(x)]^2}{x^2 g(x)}$ AND $f'(2) = 1$

$g'(2) = 2$

$f(2) = 3$

$g(2) = 1$

DERIVATIVE PROBLEM SOLUTIONS

1 a) $f'(x) = 10x - 4x^{-3} - x^{-2} \qquad = 10x - \frac{4}{x^3} - \frac{1}{x^2}$

b) $g(t) = 2t - 4t^{1/2} + 3t^{-1/3}$

$g'(t) = 2 - 2t^{-1/2} - t^{-4/3} \qquad = 2 - \frac{2}{t^{1/2}} - \frac{1}{t^{4/3}}$

c) $f'(x) = (3x^2+2)\left(5 - \frac{1}{x^2}\right) + (x^3+2x)\left(0 + 2x^{-3}\right)$

$= 15x^2 + 10 - 3 - \frac{2}{x^2} + 2 + \frac{4}{x^2}$

$= 15x^2 + \frac{2}{x^2} + 9$

d) $f'(x) = \dfrac{\left(\frac{1}{2}x^{-1/2}\right)(x^2+2) - (x^{1/2}+1)(2x)}{(x^2+2)^2}$

$= \dfrac{\frac{x^2+2}{2x^{1/2}} - \frac{(x^{1/2}+1)(2x)}{1}}{(x^2+2)^2} = \dfrac{\frac{(x^2+2) - (x^{1/2}+1)(4x^{3/2})}{2x^{1/2}}}{(x^2+2)^2}$

$= \dfrac{x^2+2 - 4x^2 - 4x^{3/2}}{2x^{1/2}} \cdot \dfrac{1}{(x^2+2)^2}$

$= \dfrac{-3x^2 - 4x^{3/2} + 2}{2x^{1/2}(x^2+2)^2}$

e) $f'(x) = 2x^2 - [(2x)(2x^2 - 3x + 1) + (x^2+1)(4x-3)]$

$= 2x^2 - [4x^3 - 6x^2 + 2x + 4x^3 + 4x - 3x^2 - 3]$

$= -8x^3 + 11x^2 - 6x + 3$

f) $g'(x) = \frac{d}{dx}\left[\frac{x}{x^2+1}\right] - \frac{d}{dx}\left[\frac{x-1}{x^2-1}\right]$

$= \frac{(1)(x^2+1)-(x)(2x)}{(x^2+1)^2} - \frac{(1)(x^2-1)-(x-1)(2x)}{(x^2-1)^2}$

$= \frac{x^2+1-2x^2}{(x^2+1)^2} - \frac{x^2-1-2x^2+2x}{(x^2-1)^2}$

$= \frac{-x^2+1}{(x^2+1)^2} - \frac{-x^2+2x-1}{(x^2-1)^2}$

$= \frac{(-x^2+1)(x^2-1)^2 - (-x^2+2x-1)(x^2+1)^2}{(x^2+1)^2(x^2-1)^2}$

g) $f'(x) = 54(-x^2+6x)^{53} \cdot \frac{d}{dx}(-x^2+6x)$

$= 54(-x^2+6x)^{53} \cdot (-2x+6)$

h) $h'(t) = \frac{(2t-2)\cdot(t+1)(t^2+1) - (t^2-2t)\cdot\frac{d}{dx}[(t+1)(t^2+1)]}{[(t+1)(t^2+1)]^2}$

$= \frac{(2t-2)(t+1)(t^2+1) - (t^2-2t)[(1)(t^2+1)+(t+1)(2t)]}{(t+1)^2(t^2+1)^2}$

$= \frac{(2t-2)(t+1)(t^2+1) - (t^2-2t)(3t^2+2t+1)}{(t+1)^2(t^2+1)^2}$

i) $$f'(x) = \frac{\frac{d}{dx}\left[(3x-1)^{1/2}\right]\cdot(x^2+1) - (3x-1)^{1/2}\cdot\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{\frac{1}{2}(3x-1)^{-1/2}(x^2+1) - (3x-1)^{1/2}(2x)}{(x^2+1)^2}$$

$$= \frac{\dfrac{x^2+1}{2(3x-1)} - \dfrac{(3x-1)^{1/2}(2x)}{1}}{(x^2+1)^2}$$

$$= \frac{(x^2+1) - (3x-1)(4x)}{2(3x-1)} \cdot \frac{1}{(x^2+1)^2}$$

$$= \frac{-11x^2+4x+1}{2(3x-1)(x^2+1)^2}$$

j) $$f'(x) = \frac{d}{dx}\left[(x^2+1)(x^2-3)\right](5x^2+x) + (x^2+1)(x^2-3)\frac{d}{dx}(5x^2+x)$$

$$= \left[(2x)(x^2-3) + (x^2+1)(2x)\right](5x^2+x) + (x^2+1)(x^2-3)(10x+1)$$

$$= (2x^3-6x)(5x^2+x) + (2x^3+2x)(5x^2+x) + (x^2+1)(x^2-3)(10x+1)$$

k) $$f'(x) = \frac{1}{2}\left((x+3)(x^2-1)\right)^{-1/2} \cdot \frac{d}{dx}\left[(x+3)(x^2-1)\right]$$

$$= \frac{1}{2\sqrt{(x+3)(x^2-1)}} \cdot \left[(1)(x^2-1) + (x+3)(2x)\right]$$

$$= \frac{x^2-1+2x^2+6x}{2\sqrt{(x+3)(x^2-1)}} = \frac{3x^2+6x-1}{2\sqrt{(x+3)(x^2-1)}}$$

2) $f'(x) = \frac{1}{2}(2x+5)^{-1/2} \cdot \frac{d}{dx}(2x+5)$

$= \frac{1}{\cancel{2}\sqrt{2x+5}} \cdot \cancel{2}$

$= \frac{1}{\sqrt{2x+5}}$

THE SLOPE OF THE TANGENT LINE AT $(2,3)$ IS GIVEN BY

$f'(2) = \frac{1}{\sqrt{2(2)+5}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

SO THE TANGENT LINE IS:

$y = ax + b$

$3 = \frac{1}{3}(2) + b$

$3 - \frac{2}{3} = b$

$\frac{7}{3} = b$

$\therefore \boxed{y = \frac{1}{3}x + \frac{7}{3}}$

3) $h'(x) = \frac{\frac{d}{dx}\left([f(x)]^2\right)x^2g(x) - [f(x)]^2\frac{d}{dx}[x^2g(x)]}{(x^2g(x))^2}$

$= \frac{2f(x)\cdot f'(x)\,x^2g(x) - [f(x)]^2[2x\,g(x) + x^2g'(x)]}{(x^2g(x))^2}$

$$h'(2) = \frac{2f(2)\cdot f'(2)\cdot(2)^2 g(2) - [f(2)]^2[2(2)g(2)+2^2g'(2)]}{(2^2 g(2))^2}$$

$$= \frac{2(3)(1)(4)(1) - (3)^2[4(1)+4(2)]}{(4(1))^2}$$

$$= \frac{24-108}{16} = -\frac{21}{4}$$