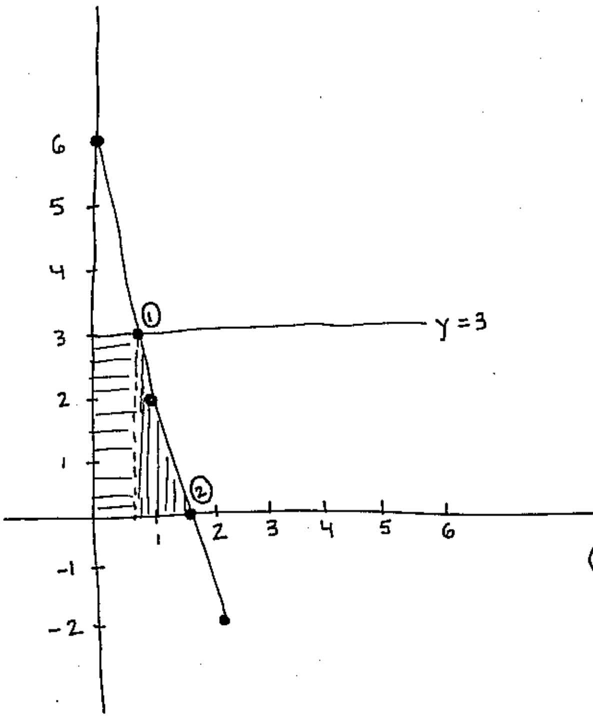
AREAS BY INTEGRATION
BONUS ASSIGNMENT
WINTER 2011
CALCULUS FOR ELECTRONICS ENGINEERING

1) $y = 6 - 4x, \quad x = 0, \quad y = 0, \quad y = 3$



① First intersection
$y = 3$ & $y = 6 - 4x$

$3 = 6 - 4x$

$-3 = -4x$

$x = 3/4 = 0.75$

② second intersection
$y = 0 \quad y = 6 - 4x$

$0 = 6 - 4x$

$4x = 6$

$x = 6/4 = 1.5$

$$\text{AREA} = \int_0^{0.75} 3 - 0 \, dx + \int_{0.75}^{1.5} (6 - 4x) - 0 \, dx$$

$$= 3x \Big|_0^{0.75} + 6x - 2x^2 \Big|_{0.75}^{1.5}$$

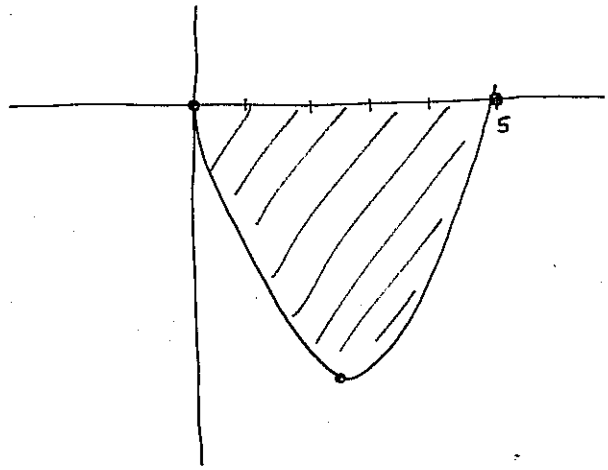
$$= 3(0.75) + \left[\left(6(1.5) - 2(1.5)^2\right) - \left(6(0.75) - 2(0.75)^2\right)\right]$$

$$= 2.25 + \left[(9 - 4.5) - (4.5 - 1.125)\right]$$

$$= 2.25 + 1.125$$

$$= \boxed{3.375}$$

#2 $y = x^2 - 5x \quad y = 0$



x-intercepts:

(y=0) $\quad 0 = x^2 - 5x$

$= x(x-5)$

$x=0 \quad x=5$

vertex $\quad x = -b/2a = 5/2 = 2.5$

$y = (2.5)^2 - 5(2.5) = -6.25$

$$\text{Area} = \int_0^5 0 - (x^2 - 5x)\,dx$$

$$= \int_0^5 -x^2 + 5x\,dx = -\frac{x^3}{3} + \frac{5x^2}{2}\Bigg|_0^5 = -\frac{5^3}{3} + \frac{5(5)^2}{2} = \boxed{\frac{125}{6}} \quad (\approx 20.833)$$

#3 $y = -x^2 + x + 2, \quad y = 0, \quad x = 0, \quad x = 3$

$y = -(x^2 - x - 2)$

$= -(x-2)(x+1)$

x-intercepts

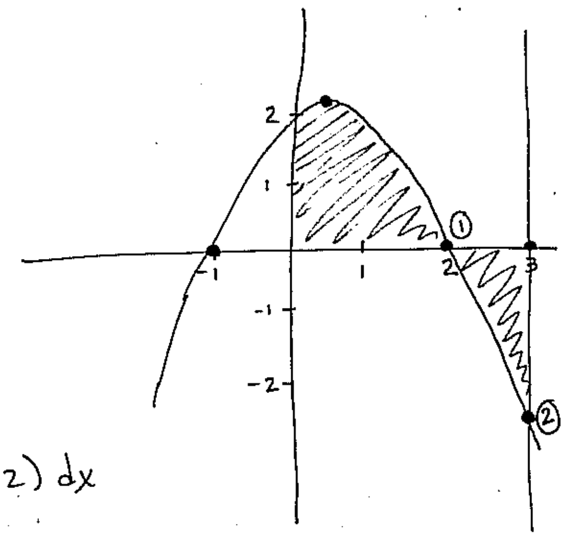
$x = 2 \quad x = -1$

vertex $\quad x = -b/2a = -1/-2 = 1/2$

$y = -(1/2)^2 + 1/2 + 2$

$= -1/4 + 2/4 + 8/4$

$= 9/4 = 2.25$



$$\text{AREA} = \int_0^2 -x^2 + x + 2\,dx + \int_2^3 0 - (-x^2 + x + 2)\,dx$$

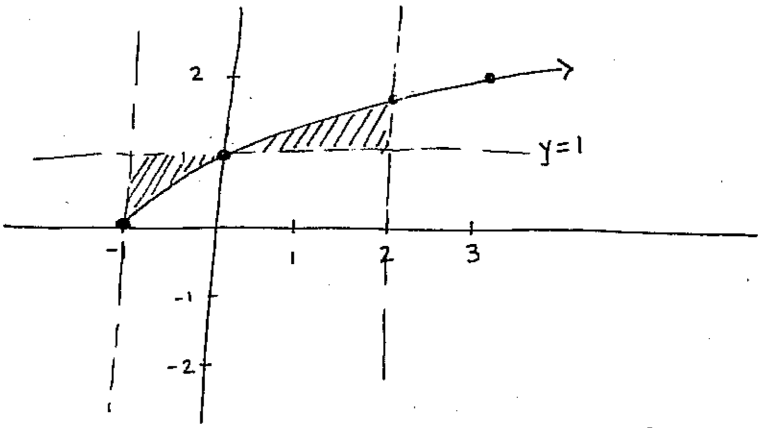
$$= -\frac{x^3}{3} + \frac{x^2}{2} + 2x\Bigg|_0^2 + \frac{x^3}{3} - \frac{x^2}{2} - 2x\Bigg|_2^3$$

$$= \left(-\frac{2^3}{3} + \frac{2^2}{2} + 2(2)\right) + \left(\left(\frac{3^3}{3} - \frac{3^2}{2} - 6\right) - \left(\frac{2^3}{3} - \frac{2^2}{2} - 4\right)\right)$$

$$= \left(-\frac{8}{3} + 2 + 4\right) + \left((9 - 4.5 - 6) - \left(\frac{8}{3} - 2 - 4\right)\right)$$

$$= -8/3 + 6 - 1.5 - 8/3 + 6 = 10.5 - 16/3 = 21/2 - 16/3 = \frac{63}{6} - \frac{32}{6} = \boxed{\frac{31}{6}} \quad (\approx 5.166)$$

#4 $\quad y=\sqrt{x+1}, \quad y=1, \quad x=-1, \quad x=2$



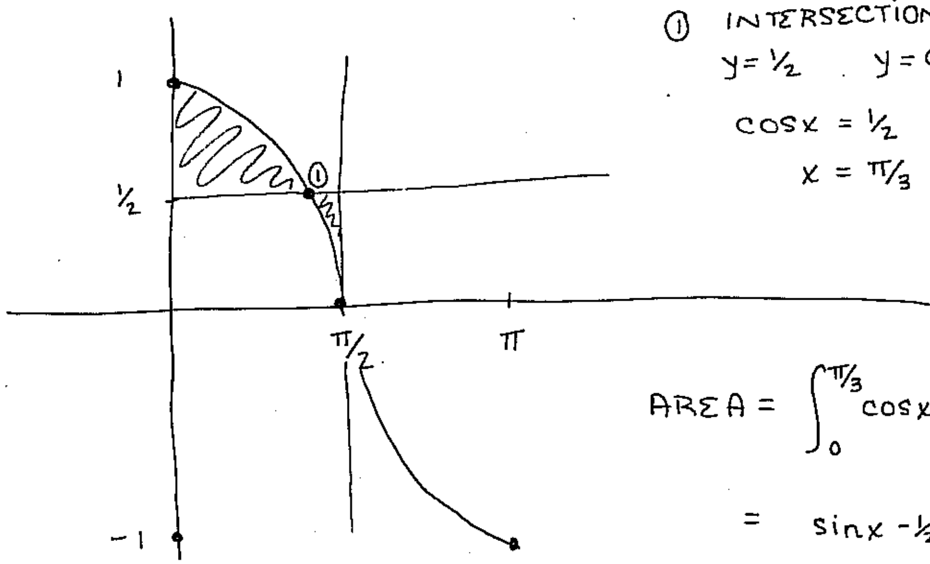
$$\text{AREA} = \int_{-1}^{0} 1-\sqrt{x+1}\,dx + \int_{0}^{2} \sqrt{x+1}-1\,dx$$

$$= x-\tfrac{2}{3}(x+1)^{3/2}\Big]_{-1}^{0} + \tfrac{2}{3}(x+1)^{3/2}-x\Big]_{0}^{2}$$

$$= \left[\left(-\tfrac{2}{3}\right)-(-1)\right] + \left[\left(\tfrac{2}{3}(3)^{3/2}-2\right)-\left(\tfrac{2}{3}\right)\right]$$

$$= \left(\tfrac{1}{3}\right] + \left[\tfrac{2}{3}(3)^{3/2}-\tfrac{8}{3}\right] = \boxed{-\tfrac{7}{3}+2\sqrt{3}} \quad (\approx 1.13)$$

#5 $\quad y=\cos x, \quad y=\frac{1}{2}, \quad x=0, \quad x=\pi/2$

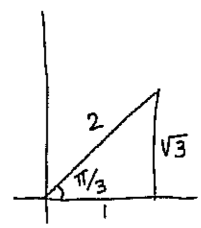


① INTERSECTION

$y=1/2 \qquad y=\cos x$

$\cos x = 1/2$

$x = \pi/3$



$$\text{AREA} = \int_{0}^{\pi/3} \cos x-\tfrac{1}{2}\,dx + \int_{\pi/3}^{\pi/2} \tfrac{1}{2}-\cos x\,dx$$

$$= \sin x-\tfrac{1}{2}x\Big]_{0}^{\pi/3} + \tfrac{1}{2}x-\sin x\Big]_{\pi/3}^{\pi/2}$$

$$= \left(\sin\pi/3-\tfrac{1}{2}(\pi/3)\right) + \left[\left(\tfrac{1}{2}\tfrac{\pi}{2}-\sin\pi/2\right)-\left(\tfrac{1}{2}\pi/3-\sin\pi/3\right)\right]$$

$$= \left(\sqrt{3}/2-\pi/6\right) + \left[\pi/4-\pi/6+\sqrt{3}/2\right]$$

$$= \sqrt{3}-\pi/6+\pi/4-\pi/6 = \boxed{\sqrt{3}-\pi/12} \quad (\approx 1.47)$$