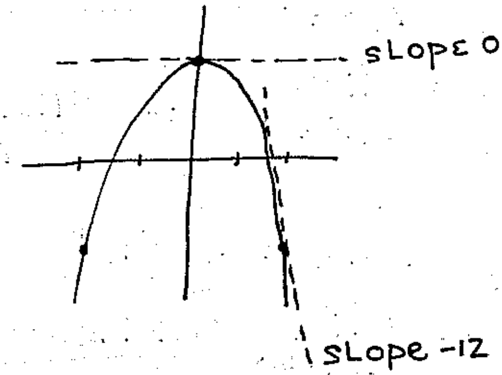
ASSIGNMENT #2
NYA FOR ELECTRO-tech
FEB 25th 2011
SOLUTIONS

P. 660

#14 $y = 4.5 - 3x^2$ AT $x=0$, $x=2$

$y' = -6x$ AT $x=0$ slope $=0$
$x=2$ slope $=-12$



$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4.5 - 3(x+\Delta x)^2 - (4.5 - 3x^2)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4.5 - 3x^2 - 6x\Delta x - 3\Delta x^2 - 4.5 + 3x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(-6x - 3\Delta x)}{\cancel{\Delta x}} = -6x$$

#18 $y = x^3 - 2x$ AT $x=-1$, $x=0$, $x=1$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

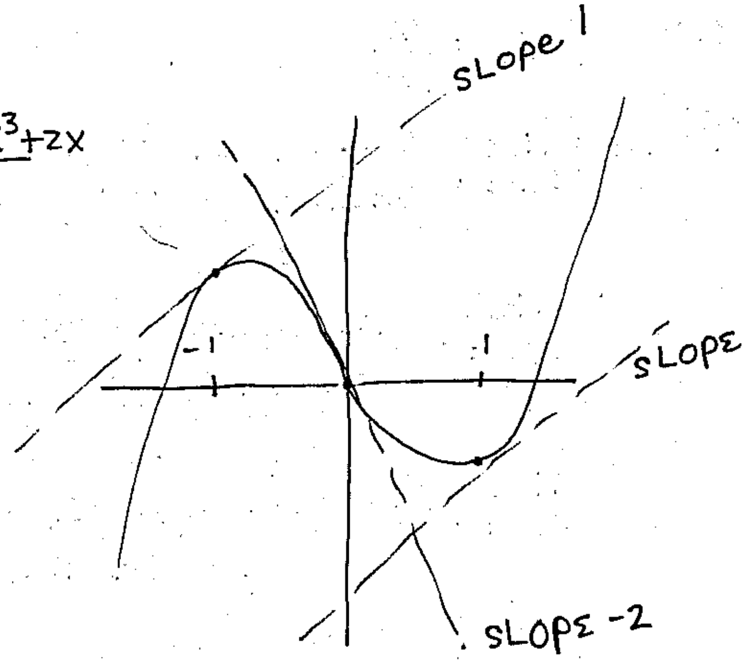
$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - 2(x+\Delta x) - (x^3 - 2x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - 2x - 2\Delta x - x^3 + 2x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(3x^2 + 3x\Delta x - 2 + \Delta x^2)}{\cancel{\Delta x}}$$

$$= 3x^2 - 2$$

AT $x=-1$ SLOPE $=1$
$x=0$ SLOPE $=-2$
$x=1$ SLOPE $=1$



P. 664

#13 $f(x) = 8x - 2x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{8(x+\Delta x) - 2(x+\Delta x)^2 - (8x - 2x^2)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{8x + 8\Delta x - 2(x^2 + 2x\Delta x + \Delta x^2) - 8x + 2x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{8\Delta x - 4x\Delta x - 2\Delta x^2}{\Delta x} = \lim_{\Delta x \to 0} 8 - 4x - 2\Delta x = \boxed{8 - 4x}$$

16 $f(x) = 2x - 4x^3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{2(x+\Delta x) - 4(x+\Delta x)^3 - (2x - 4x^3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x + 2\Delta x - 4(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3) - 2x + 4x^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2\Delta x - 4(3x^2\Delta x) - 4(3x\Delta x^2) - 4\Delta x^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2 - 12x^2 - 12x\Delta x - 4\Delta x^2)}{\cancel{\Delta x}}$$
$$= \lim_{\Delta x \to 0} 2 - 12x^2 - 12x\Delta x - 4\Delta x^2 = \boxed{2 - 12x^2}$$

20 $f(x) = \frac{x}{x-1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{x+\Delta x}{x+\Delta x - 1} - \frac{x}{x-1}}{\Delta x}$$

(PUT ON SAME DENOMINATOR)

$$= \lim_{\Delta x \to 0} \frac{\frac{(x+\Delta x)(x-1) - x(x+\Delta x - 1)}{(x+\Delta x - 1)(x-1)}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} - \cancel{x} + \cancel{x\Delta x} - \Delta x - \cancel{x^2} - \cancel{x\Delta x} + \cancel{x}}{(x+\Delta x - 1)(x-1)\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-\cancel{\Delta x}}{(x+\Delta x - 1)(x-1)(\cancel{\Delta x})} = \boxed{\frac{-1}{(x-1)^2}}$$

$\boxed{\text{P. 673}}$

\# 5 $y = x^5$

$y' = \boxed{5x^4}$

\# 6 $y = x^{12}$

$y' = \boxed{12x^{11}}$

\# 7 $f(x) = -4x^9$

$f'(x) = \boxed{-36x^8}$

\# 8 $y = -7x^6$

$y' = \boxed{-42x^5}$

\# 9 $y = 5x^4 - 3\pi$

$y' = \boxed{20x^3}$

\# 10 $s = 3t^5 + 4t$

$s' = \boxed{15t^4 + 4}$

\# 11 $y = x^2 + 2x$

$y' = \boxed{2x + 2}$

\# 12 $y = x^3 - 1.5x^2$

$y' = \boxed{3x^2 - 3x}$

\# 13 $P = 5x^3 - 2x + 1$

$P' = \boxed{15x^2 - 2}$

\# 14 $y = 6x^2 - 6x + 5$

$y' = \boxed{12x - 6}$

\# 15 $y = 25x^8 - 34x^5 - x$

$y' = \boxed{200x^7 - 170x^4 - 1}$

\# 16 $U = 4V^4 - 12V + 9$

$U' = \boxed{16V^3 - 12}$

\# 17 $f(x) = -6x^7 + 5x^3 + \pi^2$

$= \boxed{-42x^6 + 15x^2}$

\# 18 $y = 13x^4 - 6x^3 - x - 1$

$y' = \boxed{52x^3 - 18x^2 - 1}$

\# 19 $y = \frac{1}{3}x^3 + \frac{1}{2}x^2$

$y' = \boxed{x^2 + x}$

\# 20 $f(z) = -\frac{1}{4}z^8 + \frac{1}{2}z^4 - 2^3$

$f'(z) = \boxed{-2z^7 + 2z^3}$

#37

$y = 3x^2 - 6x$

$y' = 6x - 6$ SLOPE OF THE TANGENT IS 0 FOR HORIZONTAL TANGENT LINE

$0 = 6x - 6$

$= 6(x-1)$ AT $\boxed{x=1}$ TANGENT LINE IS HORIZONTAL

#38 $y = ax^2 + 2x$

$y' = 2ax + 2$

AT $x=2$ SLOPE OF TANGENT IS -4

$-4 = 2a(2) + 2$

$-6 = 4a$

$a = -6/4$ $\boxed{a = -3/2}$

#39 $y = 3x^2 - 4x$. WHEN IS $y' = 8$?

$y' = 6x - 4$

$8 = 6x - 4$

$12 = 6x$

$\boxed{x = 2}$ $\longrightarrow$ POINT ON THE CURVE $y = 3(2)^2 - 4(2)$
$= 4$

$\longrightarrow$ $\boxed{(2,4)}$

#40 $y = 5x^3 + 4x - 3$

$y' = 15x^2 + 4$ SLOPE OF TANGENT LINE IS AT LEAST 4 BECAUSE $15x^2$ IS ≥ 0 THEREFORE $15x^2 + 4 \geq 4$

#41

PERPENDICULAR TO THE LINE $x - 3y = 16$

$-3y = -x + 16$

$y = \frac{1}{3}x - 16/3$

SLOPE IS $1/3$ SO PERPENDICULAR LINE HAS SLOPE -3

$y = 2x^2 - 7x$

$y' = 4x - 7$ $\longrightarrow$ $-3 = 4x - 7$

$4 = 4x$

$x = 1$ $y = 2(1)^2 - 7(1) = -5$ $\boxed{(1,-5)}$

#43

$y = 4x^2 + 3x$ $\qquad$ $y = 5 - 2x^2$

$y' = 8x + 3$ $\qquad$ $y' = -4x$

SLOPES EQUAL:

$8x + 3 = -4x$

$12x = -3$

$x = -3/12 = \boxed{-1/4}$

$\boxed{\text{P. 682}}$

#5 $y = \sqrt{x}$

$y' = \frac{1}{2}x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}}}$

#6 $y = \sqrt[4]{x^3} = x^{3/4}$

$y' = \frac{3}{4}x^{-1/4} = \boxed{\frac{3}{4\sqrt[4]{x}}}$

#7 $V = \frac{3}{5t^2} = \frac{3}{5}t^{-2}$

$V' = -\frac{6}{5}t^{-3} = \boxed{-\frac{6}{5t^3}}$

#8 $y = \frac{2}{x^4} = 2x^{-4}$

$y' = -8x^{-5} = \boxed{\frac{-8}{x^5}}$

#9 $y = \frac{3}{\sqrt[3]{x}} = 3x^{-1/3}$

$y' = 3(-\frac{1}{3})x^{-4/3} = \boxed{\frac{-1}{x^{4/3}}}$

#10 $y = \frac{55}{\sqrt[5]{x^2}} = 55x^{-2/5}$

$y' = 55\left(\frac{-2}{5}x^{-7/5}\right) = \boxed{\frac{-22}{x^{7/5}}}$

#11 $y = x\sqrt{x} - \frac{6}{x} = x^{3/2} - 6x^{-1}$

$y' = \frac{3}{2}x^{1/2} + 6x^{-2} = \boxed{\frac{3}{2}\sqrt{x} + \frac{6}{x^2}}$

#12 $y = 2x^{-3} - 3x^{-2}$

$y' = \boxed{-6x^{-4} + 6x^{-3}}$

#13 $y = (x^2+1)^5$

$y' = 5(x^2+1)^4(2x) = \boxed{10x(x^2+1)^4}$

#14 $y = (1-2x)^4$

$y' = 4(1-2x)^3(-2) = \boxed{-8(1-2x)^3}$

#15 $y = 2.25(7-4x^3)^8$

$y' = 18(7-4x^3)^7(-12x^2)$

$= \boxed{-204x^2(7-4x^3)^7}$

#16 $s = 3(8t^2-7)^6$

$s' = 18(8t^2-7)^5(16t)$

$= \boxed{288t(8t^2-7)^5}$

#17 $y = (2x^3-3)^{1/3}$

$y' = \frac{1}{3}(2x^3-3)^{-2/3}\,6x^2$

$= \boxed{2x^2(2x^3-3)^{-2/3}}$

#18 $y = 8(1-6x)^{1.5}$

$$y' = 12(1-6x)^{0.5}(-6)$$
$$= \boxed{-72(1-6x)^{0.5}}$$

P. 696

#5 $y = \frac{1}{x^2+1}$ at $(1, \frac{1}{2})$

$$= (x^2+1)^{-1} \qquad m = y' = -(x^2+1)^{-2}(2x)$$

AT $x=1 \Rightarrow -(2)^{-2}(2) = \frac{-2}{4} = -\frac{1}{2}$

$$y = -\tfrac{1}{2}x + b$$
$$\tfrac{1}{2} = -\tfrac{1}{2}(1) + b$$
$$b = 1$$

EQUATION $\boxed{y = -\frac{1}{2}x + 1}$

#11 $y = x^2 - 2x$

$y' = 2x - 2$ with slope 2

$$2 = 2x - 2$$
$$4 = 2x$$
$$x = 2$$

$y = 2^2 - 2(2) = 0$ point $(2, 0)$

$$y = mx + b$$
$$y = 2x + b$$
$$0 = 2(2) + b$$
$$b = -4$$

$\boxed{y = 2x - 4}$

#14 $y = \frac{1}{2}x^4 + 1$ NORMAL line w/ slope 4

$y' = 2x^3$ NEGATIVE reciprocal: $-\frac{1}{4}$

$$-\tfrac{1}{4} = 2x^3$$
$$-\tfrac{1}{8} = x^3$$
$$x = -\tfrac{1}{2}$$

$y = \frac{1}{2}(-\frac{1}{2})^4 + 1 = \frac{1}{32} + 1 = \frac{33}{32}$

$$y = 4x + b$$

$\frac{33}{32} = 4(-\frac{1}{2}) + b \longrightarrow b = \frac{33}{32} + 2 = \frac{97}{32}$ $\boxed{y = 4x + \frac{97}{32}}$

P. 801

#3 $Y = \sin(x+2)$

$\boxed{Y' = \cos(x+2)}$

#4 $Y = 3\sin 4x$

$Y' = 3(\cos 4x)(4)$

$= \boxed{12\cos 4x}$

#5 $Y = 2\sin(2x^3-1)$

$Y' = 2\cos(2x^3-1)(6x^2)$

$= \boxed{12x^2\cos(2x^3-1)}$

#6 $S = 5\sin(7-3t)$

$S' = 5\cos(7-3t)(-3)$

$= \boxed{-15\cos(7-3t)}$

#18 $Y = 0.5\theta\cos(2\theta+\pi/4)$

$Y' = 0.5\cos(2\theta+\frac{\pi}{4}) - \sin(2\theta+\pi/4)(2)(0.5\theta)$

$= \boxed{0.5\cos(2\theta+\pi/4) - \theta\sin(2\theta+\frac{\pi}{4})}$

#20 $Y = 6\sin x\cos 4x$

$Y' = 6\cos x\cos 4x - \sin 4x(4)\sin x(6)$

$= \boxed{6\cos x\cos 4x - 24\sin 4x\sin x}$

#22 $Y = (x-\cos^2 x)^4$

$Y' = 4(x-\cos^2 x)^3(1-2\cos x(-\sin x))$

$= \boxed{4(x-\cos^2 x)(1+2\cos x\sin x)}$

#24 $T = \dfrac{4z+3}{\sin\pi z}$

$T' = \dfrac{4(\sin\pi z) - \cos\pi z(\pi)(4z+3}{\sin^2\pi z}$

$= \boxed{\dfrac{4\sin\pi z - \pi(4z+3)\cos\pi z}{\sin^2\pi z}}$

#26 $Y = \dfrac{\cos^2 3x}{1+2\sin^2 2x}$

$Y' = \boxed{\dfrac{\left[2\cos 3x(-\sin 3x)(3)(1+2\sin^2 2x) - (4\sin 2x\cos 2x(2))\cos^2 3x\right]}{(1+2\sin^2 2x)^2}}$

#28 $Y = \cos^3 4x\sin^2 2x$

$Y' = \boxed{3\cos^2 4x(-\sin 4x)4\sin^2 2x + 2\sin 2x\cos 2x(2)\cos^3 4x}$

#30 $z = 0.2\cos(\sin 3\phi)$

$\boxed{z' = 0.2(-\sin(\sin 3\phi))(\cos 3\phi)3}$

#32 $Y = x\sin x + \cos x$

$Y' = \sin x + (\cos x)x - \sin x$

$= \boxed{x\cos x}$

P. 805

#3 $y = \tan 5x$

$\boxed{y' = (\sec^2 5x)5}$

#4 $y = 3\tan(3x+2)$

$\boxed{y' = 3\sec^2(3x+2)(3)}$

#5 $y = 5\cot(0.25\pi - \theta)$

$= \dfrac{5}{\tan(0.25\pi-\theta)} = 5(\tan(0.25\pi-\theta))^{-1}$

$y' = -5(\tan(0.25\pi-\theta))^{-2}\sec^2(0.25\pi-\theta)(-1)$

$= \boxed{\dfrac{5\sec^2(0.25\pi-\theta)}{(\tan(0.25\pi-\theta))^2}}$

P. 816

#7 $u = 8\ln(3-x)$

$u' = \boxed{\dfrac{8}{3-x}(-1)}$

#8 $y = 2\ln(3x^2-1)$

$\boxed{y' = \dfrac{2}{3x^2-1}(6x)}$

#9 $y = 2\ln\tan 2x$

$y' = \boxed{\dfrac{2}{\tan 2x}(\sec^2 2x)(2)}$

#10 $s = \ln\sin^2 t$

$s' = \boxed{\dfrac{1}{\sin^2 t}(2\sin t\cos t)}$

#11 $R = \ln\sqrt{T} = \frac{1}{2}\ln T$

$\boxed{R' = \dfrac{1}{2T}}$

#12 $y = \ln(4x-3)^3$

$y' = \dfrac{1}{(4x-3)^3}3(4x-3)^2(4)$

$= \boxed{\dfrac{12}{4x-3}}$

#13 $y = \ln(x-x^2)^3$ use log properties then derive

$y = 3\ln(x-x^2)$

$y' = \boxed{\dfrac{3}{x-x^2}(1-2x)}$

#14 $s = 3\ln^2(7t^3-1)$

$= 3[\ln(7t^3-1)]^2$

$s' = 6\ln(7t^3-1)\left(\dfrac{1}{7t^3-1}\right)(21t^2)$

#15 $v = 3(t+\ln t^2)^2$

$v' = 6(t+\ln t^2)\left(1+\dfrac{1}{t^2}(2t)\right)$

$= \boxed{6(t+\ln t^2)(1+\frac{2}{t})}$

#16 $y = 6x^2 \ln 5x$

$y' = 12x \ln 5x + \frac{1}{5x}(5)\,6x^2$

$= \boxed{12x \ln 5x + 6x}$

#17 $y = 3x \ln(6-x)$

$y' = 3\ln(6-x) + \frac{1}{6-x}(-1)(3x)$

$= \boxed{3\ln(6-x) - \frac{3x}{6-x}}$

P. 819

#4 $y = 10^{x^2}$

$y' = \boxed{\ln 10 \; 10^{x^2}(2x)}$

#6 $r = 0.3e^{\theta^2}$

$\boxed{r' = 0.3e^{\theta^2}(2\theta)}$

#8 $y = 0.6\ln(e^{5x}+3)$

$y' = \boxed{\frac{0.6}{e^{5x}+3}(e^{5x}\cdot 5)}$

#10 $y = 5x^2 e^{2x}$

$\boxed{y' = 10xe^{2x} + e^{2x}(2)5x^2}$

#12 $y = 4e^x \sin \tfrac{1}{2}x$

$y' = 4e^x \sin\tfrac{1}{2}x + \cos\tfrac{1}{2}x\,(\tfrac{1}{2})4e^x$

$= \boxed{2e^x(2\sin\tfrac{1}{2}x + \cos\tfrac{1}{2}x)}$

#18 $y = \frac{7\ln 3x}{e^{2x}+8}$

$y' = \frac{\frac{7}{3x}\cdot 3(e^{2x}+8) - e^{2x}(2)(7\ln 3x)}{(e^{2x}+8)^2}$

$= \boxed{\frac{21(e^{2x}+8) - 42xe^{2x}\ln 3x}{3x(e^{2x}+8)^2}}$

P. 825

#17 $y = x^2 \ln x$ AT $(1,0)$

$y' = 2x\ln x + \frac{1}{x}x^2$

AT $x=1$

$y' = 2\ln 1 + 1$

$= 1$

$y = x + b$

$0 = 1 + b \quad b = -1$

EQUATION: $\boxed{y = x - 1}$