ASSIGNMENT #3
NYA ELECTROTECH
DAWSON COLLEGE
MARCH 11th 2011

TOTAL: 30 MARKS

Section 23.8

6. $x^5 - 5y = 6 - x$

$5x^4 - 5y' = -1$

$-5y' = -1 - 5x^4$

$y' = \frac{1}{5} + x^4$

(1 MARK)

14. $8y - xy - 7 = 0$

$8y' - (y + y'x) = 0$

$8y' - xy' = y$

$y' = \frac{y}{8-x}$

(1 Mark)

16. $y^2x - \frac{5y}{x+1} + 3x = 4$

$2yy'x + y^2 - \left[\frac{5y'(x+1) - 5y}{(x+1)^2}\right] + 3 = 0$

$2yy'x(x+1)^2 + y^2(x+1)^2 - 5y'(x+1) + 5y + 3(x+1)^2 = 0$

$\left[2xy(x+1)^2 - 5(x+1)\right]y' = -y^2(x+1)^2 - 5y - 3(x+1)^2$

$y' = -\frac{y^2(x+1)^2 - 5y - 3(x+1)^2}{2xy(x+1)^2 - 5(x+1)}$

(2 MARKS)

29. $x^2 + y^2 = 4x$

$2x + 2yy' = 4$

$2yy' = 4 - 2x$

$y' = \frac{4-2x}{2y}$

HORIZONTAL TANGENT WHEN $y' = 0$

$\Rightarrow x = 2$

$y^2 = 4x - x^2$

$y^2 = 4(2) - 2^2$

$y^2 = 4$

$y = \pm 2$

Two points : (2,2) & (2,-2) (2 MARKS)

31. $xy + y^2 + 2 = 0$ SLOPE OF TANGENT at $(-3,1)$

$y + y'x + 2yy' = 0$

$y' = \frac{-y}{x+2y} \longrightarrow y' = \frac{-1}{-3+2(1)} = \frac{-1}{-1} = \boxed{1}$

(2 marks)

32. $2x^2 + y^2 = 24$ & $y^2 = 8x$ perpendicular at $(2,4)$

$4x + 2yy' = 0$ $\qquad 2yy' = 8$

$y' = \frac{-4x}{2y}$ $\qquad y' = \frac{4}{y}$

AT $(2,4)$ $\qquad$ AT $(2,4)$

$y' = \frac{-4(2)}{2(4)} = -1$ $\qquad y' = \frac{4}{4} = 1$

Slopes -1 & 1 are negative reciprocals of each other at $(2,4)$ thus tangent lines are perpendicular there (2 marks)

SECTION 27.1

41. $\sin(xy) + \cos(2y) = x^2$

$(\cos xy)(+xy') - (\sin 2y)(2y') = 2x$

$y\cos xy + x(\cos xy)y' - (2\sin 2y)y' = 2x$

$$\boxed{y' = \frac{2x - y\cos xy}{x\cos xy - 2\sin 2y}}$$

(2 marks)

42. $x\cos 2y + \sin x \cos y = 1$

$\cos 2y - x\sin 2y(2y') + \cos x\cos y - (\sin y)y'\sin x = 0$

$-2x(\sin 2y)y' - (\sin x)(\sin y)y' = -\cos x\cos y - \cos 2y$

$$\boxed{y' = \frac{\cos x\cos y + \cos 2y}{2x\sin 2y + \sin x\sin y}}$$

(2 marks)

33. $x\sec y - 2y = \sin 2x$

$$\frac{x}{\cos y} - 2y = \sin 2x$$

$$\frac{\cos y + (x\sin y)y'}{\cos^2 y} - 2y' = (\cos 2x)2$$

$$\cos y + (x\sin y)y' - 2\cos^2 y\, y' = (\cos 2x)(\cos^2 y)2$$

$$\boxed{y' = \frac{2\cos 2x\cos^2 y - \cos y}{x\sin y - 2\cos^2 y}} \quad (2 \text{ MARKS})$$

34. $3\cot(x+y) = \cos y^2$

$$\frac{3}{\tan(x+y)} = \cos y^2$$

$$3 = \tan(x+y)\cos y^2$$

$$0 = \sec^2(x+y)(1+y')\cos y^2 - \sin y^2\, 2yy' + \tan(x+y)$$

$$0 = \sec^2(x+y)\cos y^2 + \sec^2(x+y)\cos y^2\, y' - \sin y^2(2yy') + \tan(x+y)$$

$$2y\sin y^2 + \tan(x+y)y' - \sec^2(x+y)\cos y^2 = \sec^2(x+y)\cos y^2$$

$$\boxed{y' = \frac{\sec^2(x+y)\cos y^2}{2y\sin y^2 + \tan(x+y) - \sec^2(x+y)\cos y}} \quad (2 \text{ MARKS})$$

SECTION 27.3

32 $\sin^{-1}(x+y) + y = x^2$

$$\frac{1}{\sqrt{1-(x+y)^2}}(1+y') + y' = 2x$$

$$1 + y' + \sqrt{1-(x+y)^2}\,y' = \sqrt{1-(x+y)^2}\,(2x)$$

$$\boxed{y' = \frac{2x\sqrt{1-(x+y)^2} - 1}{1 + \sqrt{1-(x+y)^2}}} \quad (2 \text{ MARKS})$$

33. $2\tan^{-1}(xy) + x = 3$

$$\frac{2}{1+(xy)^2}(+xy') + 1 = 0$$

$$2(y + xy') = -(y+(xy)^2)$$

$$2xy' = -y - (xy)^2 - 2y$$

$$\boxed{y' = \frac{-3-(xy)^2}{2x}}$$

(2 marks)

33. $y = x - \ln^2(x+y)$

$$y' = 1 - 2\ln(x+y)\frac{1}{x+y}(1+y')$$

$$y'(x+y) = x+y - 2\ln(x+y)(1+y')$$

$$y'(x+y) + 2\ln(x+y)y' = x+y-2\ln(x+y)$$

$$\boxed{y' = \frac{x+y-2\ln(x+y)}{x+y+2\ln(x+y)}}$$ (2 marks)

SECTION 27.6

25. $y = xe^{xy} + \sin y$

$$y' = e^{xy} + xe^{xy}(y+xy') + (\cos y)y'$$

$$y' = e^{xy} + xye^{xy} + x^2e^{xy}y' + (\cos y)y'$$

$$y' - x^2e^{xy}y' - \cos y\, y' = e^{xy} + xye^{xy}$$

$$\boxed{y' = \frac{e^{xy} + xye^{xy}}{1 - x^2e^{xy} - \cos y}}$$

(2 MARKS)

SECTION 27.5

32. $3\ln xy + \sin y = x^2$

$$\frac{3}{xy}(y+xy') + (\cos y)y' = 2x$$

$$3y + 3xy' + xy(\cos y)y' = 2x^2y$$

$$\boxed{y' = \frac{2x^2y - 3y}{3x + xy\cos y}}$$

(2 marks)

26. $y = 4e^{-2x^{-1}}\ln y + 1$

$$y' = 4e^{-2x^{-1}}(2x^{-2})\ln y + \frac{1}{y}y'4e^{-2x^{-1}}$$

$$y' = \frac{8e^{-2/x}}{x^2}\ln y + \frac{4}{y}e^{-2/x}y'$$

$$x^2y\,y' = 8ye^{-2/x}\ln y + 4x^2e^{-2/x}y'$$

$$\boxed{y' = \frac{8ye^{-2/x}\ln y}{x^2y - 4x^2e^{-2/x}}}$$

(2 MARKS)