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ASSIGNMENT #4
CALCULUS I (ELECTROTECH)
WINTER 2011
SOLUTIONS

(SECTION 23.9)

12. $y = 6x - 2x^5$

$y' = 6 - 10x^4$

$y'' = \boxed{-40x^3}$

(1 mark)

26. $y = \frac{x}{\sqrt{2x}}$

$= \frac{x}{\sqrt{2}\sqrt{x}} = \frac{1}{\sqrt{2}} x^{1/2}$

$y' = \frac{1}{2\sqrt{2}} x^{-1/2}$

$y'' = \boxed{\frac{-1}{4\sqrt{2}} x^{-3/2}}$

(2 marks)

(SECTION 27.1)

44. $y = \cos 2x$

$y' = -(\sin 2x)(2)$

$y'' = -2 \cos 2x (2)$

$= -4 \cos 2x$

$= -4y$

(2 marks)

14. $r = 3\theta^2 - \frac{1}{2\sqrt{\theta}}$

$= 3\theta^2 - \frac{1}{2}\theta^{-1/2}$

$r' = 6\theta + \frac{1}{4}\theta^{-3/2}$

$r'' = \boxed{6 - \frac{3}{8}\theta^{-5/2}}$

(2 marks)

30. $xy = y^2 + 2e^3$

$y + xy' = 2yy'$

$xy' - 2yy' = -y$

$y'(x-2y) = -y$

$y' = \frac{-y}{x-2y}$

$y'' = \frac{-y'(x-2y) - (1-2y')(-y)}{(x-2y)^2}$

$= \frac{y - (1 + \frac{2y}{x-2y})(-y)}{(x-2y)^2}$

SUBSTITUTE
 $y' = \frac{-y}{x-2y}$

$= \frac{y}{(x-2y)^2} + \frac{y}{(x-2y)^2} + \frac{2y^2}{(x-2y)^3}$

(3 marks)

(SECTION 27.5)

47. $y = x^x$

$\ln y = \ln x^x$

$\ln y = x \ln x$

$\frac{1}{y} y' = \ln x + x \left(\frac{1}{x}\right) \rightarrow y' = \boxed{x^x (\ln x + 1)}$

(2 marks)

48. $y = (\sin x)^x$

$\ln y = \ln(\sin x)^x$

$\ln y = x \ln \sin x$

$\frac{1}{y} y' = \ln \sin x + \frac{1}{\sin x} \cos x (x)$

$y' = y (\ln \sin x + x \cot x)$

$y' = \boxed{(\sin x)^x (\ln \sin x + x \cot x)}$

(2 marks)

56. $V = k \ln \left(\frac{\sqrt{a^2+x^2} + a}{\sqrt{a^2+x^2} - a} \right)$

$= k \left(\ln(\sqrt{a^2+x^2} + a) - \ln(\sqrt{a^2+x^2} - a) \right)$

$E = -V'(x) = -k \left(\frac{1}{\sqrt{a^2+x^2} + a} \left(\frac{1}{2} (a^2+x^2)^{-1/2} \cdot 2x \right) - \frac{1}{\sqrt{a^2+x^2} - a} \left(\frac{1}{2} (a^2+x^2)^{-1/2} \cdot 2x \right) \right)$

$= -k \left[\frac{x}{a^2+x^2 + a\sqrt{a^2+x^2}} - \frac{x}{a^2+x^2 - a\sqrt{a^2+x^2}} \right]$

$= -k \left[\frac{xa^2+x^3 - ax\sqrt{a^2+x^2} - xa^2 - x^3 - ax\sqrt{a^2+x^2}}{(a^2+x^2 + a\sqrt{a^2+x^2})(a^2+x^2 - a\sqrt{a^2+x^2})} \right]$

$= -k \left[\frac{-2ax\sqrt{a^2+x^2}}{(a^2+x^2)^2 - a^2(a^2+x^2)} \right] = -k \left[\frac{-2ax\sqrt{a^2+x^2}}{(a^2+x^2)(a^2+x^2 - a^2)} \right]$

$= -k \left[\frac{-2ax\sqrt{a^2+x^2}}{(a^2+x^2)x^2} \right] = -k \left[\frac{-2a}{x\sqrt{a^2+x^2}} \right] = \frac{2ka}{x\sqrt{a^2+x^2}}$

(2 marks)

(SECTION 24.5)

6. $y = 2 + 6x - 3x^2$

$y' = 6 - 6x$

$= 6(1-x)$

CRITICAL pts $x=1$

INTERVALS $(-\infty, 1)$ $(1, \infty)$

TEST pt 0 2

sign of f' + -

BEHAVIOUR ↗ ↘

(2 marks)

y is INCREASING on $(-\infty, 1)$
 y is DECREASING on $(1, \infty)$

8. $y = x^4 - 6x^2$

$y' = 4x^3 - 12x$

$= 4x(x^2 - 3)$

CRITICAL pts $x=0, x=\pm\sqrt{3}$

INTERVALS $(-\infty, -\sqrt{3})$ $(-\sqrt{3}, 0)$ $(0, \sqrt{3})$ $(\sqrt{3}, \infty)$

TEST pt -3 -1 1 3

sign of f' - + - +

BEHAVIOUR ↘ ↗ ↘ ↗

y is INCREASING ON $(-\sqrt{3}, 0)$ & $(\sqrt{3}, \infty)$
 y is DECREASING ON $(-\infty, -\sqrt{3})$ & $(0, \sqrt{3})$

(2 marks)

(SECTION 24.5)

14. $y = 2 + 6x - 3x^2$

$y' = 6 - 6x$

$y'' = -6$

y'' is ALWAYS NEGATIVE

y is CONCAVE down on $(-\infty, \infty)$ (1 mark)

16. $y = x^4 - 6x^2$
 $y' = 4x^3 - 12x$
 $y'' = 12x^2 - 12$
 $= 12(x^2 - 1)$

CRITICAL PTS $x = \pm 1$

INTERVALS	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
TEST	-2	0	2
SIGN OF f''	+	-	+
CONCAVITY	U	∩	U

INFLECTION PTS. AT

$x = -1$	$y = -5$	$(-1, -5)$
$x = 1$	$y = -5$	$(1, -5)$

(3 marks)

24.

$y = x^3 - 9x^2 + 15x + 1$
 $y' = 3x^2 - 18x + 15$
 $= 3(x^2 - 6x + 5)$
 $= 3(x-1)(x-5)$

CRITICAL PTS
 $x=1$ $x=5$

INTERVALS	$(-\infty, 1)$	$(1, 5)$	$(5, \infty)$
TEST PT	0	2	6
SIGN OF f'	+	-	+
BEHAVIOUR	↗	↘	↗

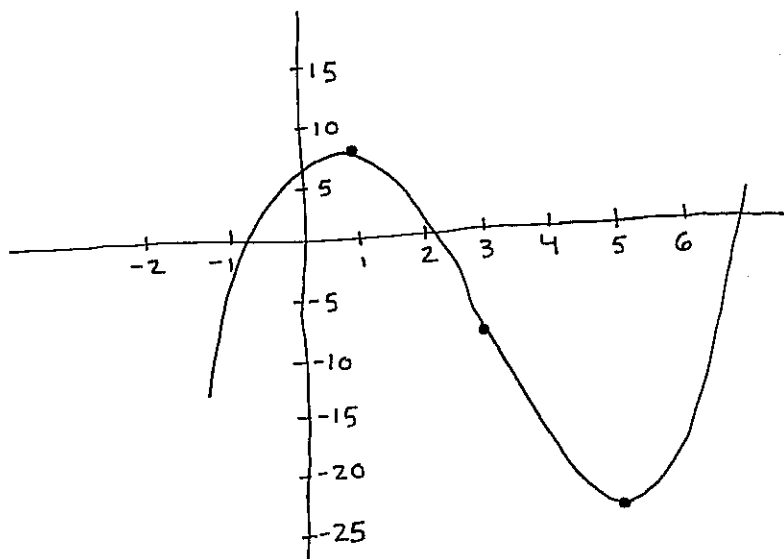
MAX AT $x=1$ $y=8$
 MIN AT $x=5$ $y=-24$

$y'' = 6x - 18$
 $= 6(x-3)$

CRITICAL PTS
 $x=3$

INTERVALS	$(-\infty, 3)$	$(3, \infty)$
TEST PT	0	4
SIGN OF f''	-	+
CONCAVITY	∩	U

INFLECTION PT AT $x=3$
 $y=-8$



(5 marks)

26. $y = x^3 - 12x + 12$

$y' = 3x^2 - 12$
 $= 3(x^2 - 4)$ CRITICAL PTS ± 2

$y'' = 6x$ CRITICAL PTS 0

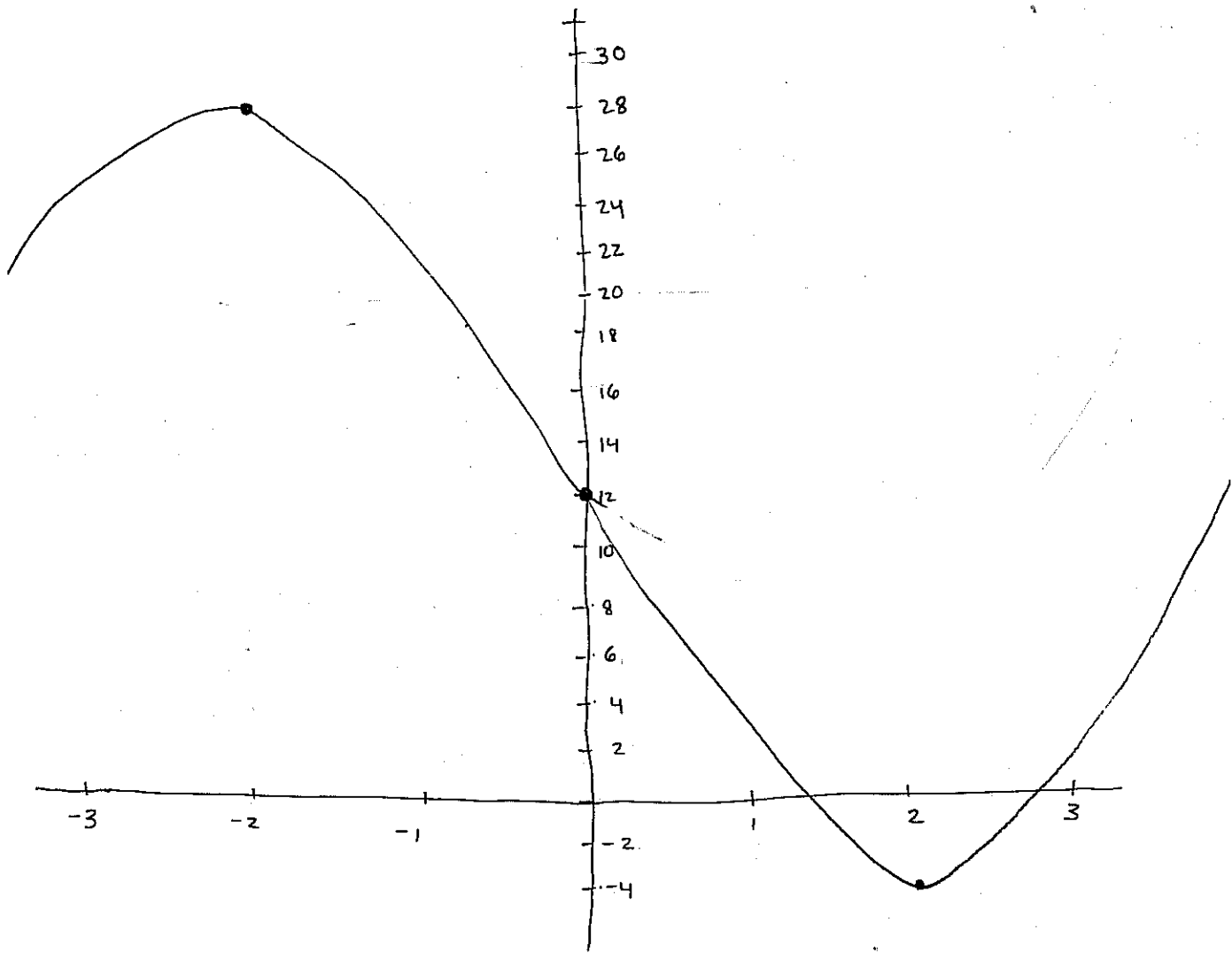
INTERVAL	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
TEST	-3	0	3
SIGN y'	+	-	+
BEHAVIOUR	↗	↘	↗

MAX AT $x=-2$ $y=28$
 MIN AT $x=2$ $y=-4$

INTERVAL	$(-\infty, 0)$	$(0, \infty)$
TEST	-1	1
SIGN y''	-	+
CONCAVITY	∩	U

INFLECTION pt at $x=0$ $y=12$

(5 marks)



28. SEE CLASS NOTES.
(4 marks)