SOLUTIONS (Assignment #5)

NYA ELECTRO
MAY 2ND 2011

Find the ANTIDERIVATIVES

#14 $f(x) = \frac{4}{3} x^{1/3}$

$\boxed{F(x) = x^{4/3} + C}$

#16 $f(x) = 12x^5 + 2x$

$\boxed{F(x) = 2x^6 + x^2 + C}$

#18 $f(x) = x^2 - 5$

$\boxed{F(x) = \frac{1}{3}x^3 - 5x + C}$

#20 $f(s) = 9\sqrt[3]{s} + 3$

$\boxed{F(s) = \frac{27}{4} s^{4/3} + 3s + C}$

#20 $f(x) = 8/x^5 = 8x^{-5}$

$\boxed{F(x) = -2x^{-4} + C}$

#24 $f(x) = \frac{1}{2\sqrt{x}} + \sqrt{3}$

$\boxed{F(x) = x^{1/2} + \sqrt{3}\,x + C}$

#26 $f(x) = x\sqrt{x} - x^{-3}$
$= x^{3/2} - x^{-3}$

$\boxed{F(x) = \frac{2}{5}x^{5/2} + \frac{1}{2}x^{-2} + C}$

(7 MARKS)

Find the integrals (SECTION 25.2)

#6 $\int 5x^4\,dx = \boxed{x^5 + C}$

#10 $\int 6\sqrt[3]{x}\,dx = \boxed{\frac{18}{4}x^{4/3} + C}$

#12 $\int \frac{4}{\sqrt{x}}\,dx = \boxed{8x^{1/2} + C}$

#22 $\int x^{1/3} + x^{1/5} + x^{-1/7}\,dx$

$= \boxed{\frac{3}{4}x^{4/3} + \frac{5}{6}x^{6/5} + \frac{7}{6}x^{6/7} + C}$

#24 $\int (x^2+4x+4)^{1/3}\,dx$

$= \int ((x+2)^2)^{1/3}\,dx$

$= \int (x+2)^{2/3}\,dx$

$= \boxed{\frac{3}{5}(x+2)^{5/3}+C}$

(5 marks (1 mark each))

#53 $\frac{di}{dt} = 4t - 0.6t^2$

$i = 2A$ when $t = 0s$

$i = \int 4t - 0.6t^2\,dt$

$i = 2t^2 - 0.2t^3 + C$

$2 = 2(0)^2 - 0.2(0)^3 + C$

$C = 2$

$\boxed{i = 2t^2 - 0.2t^3 + 2}$

(2 marks)

SECTION 28.1

#3 $\int \sin^4 x \cos x\,dx$

$U = \sin x$
$dU = \cos x\,dx$

$= \int U^4\,du = \frac{U^5}{5} + C$

$= \boxed{\frac{1}{5}\sin^5 x + C}$

#6 $\int 8\sin^{1/3} x \cos x\,dx$

$U = \sin x$
$dU = \cos x$

$= \int 8U^{1/3}\,du = 8\frac{U^{4/3}}{4/3} + C$

$= \boxed{6\sin^{4/3} x + C}$

(8 marks)

#5 $\int 0.4\sqrt{\cos\theta}\,\sin\theta\,d\theta$

$U = \cos\theta$
$du = -\sin\theta\,d\theta$
$-du = \sin\theta\,d\theta$

$= \int -0.4\sqrt{U}\,du$

$= -0.4\frac{U^{3/2}}{3/2} + C$

$= \frac{-4}{15}U^{3/2} + C = \boxed{\frac{-4}{15}(\cos\theta)^{3/2} + C}$

#7 $\int 4\tan^2 x \sec^2 x\,dx$

$U = \tan x$
$dU = \sec^2 x\,dx$

$= \int 4U^2\,du = \frac{4U^3}{3} + C$

$= \boxed{\frac{4}{3}\tan^3 x + C}$

p. 838 SECTION 28.2

#4 $\int \frac{1}{-4x+1}\,dx$

$u = 1-4x$
$du = -4\,dx$

$= \int \frac{1}{u}\left(-\frac{1}{4}\right) du$

$= -\frac{1}{4} \ln|u| + C$

$= \boxed{-\frac{1}{4} \ln|1-4x| + C}$

#6 $\int \frac{4\sqrt{u}}{1+u\sqrt{u}}\,du$

$x = 1+u\sqrt{u}$
$x = 1+u^{3/2}$
$dx = \frac{3}{2} u^{1/2}\,du$

$2/3\,dx = \sqrt{u}\,du$
$8/3\,dx = 4\sqrt{u}\,du$

$= \int 8/3 \frac{1}{x}\,dx = \frac{8}{3} \ln|\;| + C$

$= \boxed{8/3 \ln|1+u\sqrt{u}| + C}$

#10 $\int \frac{\sin 3x}{\cos 3x}\,dx$

$u = \cos 3x$
$du = -3 \sin 3x\,dx$
$-\frac{1}{3}\,du = \sin 3x\,dx$

$= \int -\frac{1}{3} \frac{1}{u}\,du$

$= -1/3 \ln|u| + C$

$= \boxed{-1/3 \ln|\cos 3x| + C}$

(6 marks)

p. 841 SECTION 28.3

#4 $\int 4e^{x^4} x^3\,dx$

$u = x^4 \qquad du = 4x^3\,dx$

$= \int e^u\,du = e^u + C$

$= \boxed{e^{x^4} + C}$

#14 $\int \sec^2 x\, e^{\tan x}\,dx$

$u = \tan x$
$du = \sec^2 x\,dx$

$= \int e^u\,du = e^u + C$

$= \boxed{e^{\tan x} + C}$

#16 $\int \frac{4}{x^2 e^{1/x}}\,dx$

$u = 1/x \qquad du = -\frac{1}{x^2}\,dx$

$= \int -\frac{4}{e^u}\,du = \int -4e^{-u}\,du$

$= 4e^{-u} + C = \boxed{4e^{-1/x} + C}$

#19 $\displaystyle\int \frac{2}{\sqrt{x}\, e^{\sqrt{x}}}\, dx$

SECTION 25.2 p.742

$U = \sqrt{x}$

$du = \frac{1}{2} x^{-1/2} dx$

$du = \frac{1}{2\sqrt{x}}\, dx$

$4\, du = \frac{2}{\sqrt{x}}\, dx$

$= \displaystyle\int \frac{4}{e^{U}}\, du$

$= \displaystyle\int 4e^{-U}\, du$

$= -4e^{-u} + C$

$= \boxed{-4e^{-\sqrt{x}} + C}$ (8 marks)

#28 $\displaystyle\int (t^3-2)^6\, 3t^2\, dt$

$U = t^3 - 2$

$dU = 3t^2\, dt$

$= \displaystyle\int U^6\, du = \frac{U^7}{7} + C$

$= \boxed{\frac{1}{7}(t^3-2)^7 + C}$

#30 $\displaystyle\int 6x^2(1-x^3)^{4/3}\, dx$

$U = 1 - x^3$

$dU = -3x^2\, dx$

$-2\, dU = 6x^2\, dx$

$= \displaystyle\int -2U^{4/3}\, du = -\frac{2U^{7/3}}{7/3} + C$

$= \boxed{-\frac{6}{7}(1-x^3)^{7/3} + C}$

#32 $\displaystyle\int \frac{dv}{(0.3+2v)^3}$

$U = 0.3 + 2V$

$du = 2\, dV$

$\frac{1}{2}\, du = dV$

$= \displaystyle\int \frac{1}{2} U^{-3}\, du$

$= \frac{1}{2}\frac{U^{-2}}{-2} + C$

$= \boxed{-\frac{1}{4}(0.3+2V)^{-2} + C}$

#36 $\displaystyle\int (x^2-x)\left(x^3 - \frac{3}{2}x^2\right)^8 dx$

$U = x^3 - \frac{3}{2}x^2$

$dU = 3x^2 - 3x\, dx$

$dU = 3(x^2-x)\, dx$

$\frac{1}{3}dU = (x^2-x)\, dx$

$= \displaystyle\int \frac{1}{3} U^8\, du = \frac{1}{3}\frac{U^9}{9} + C$

$= \boxed{\frac{1}{27}\left(x^3 - \frac{3}{2}x^2\right)^9 + C}$

(8 marks)

SECTION 26.1

#19 $i = 0.06t\sqrt{1+t^2}$

At $t=0$ $\quad Q = 0.015\,C$

$$Q = \int i \, dt$$

$$= \int 0.06t\sqrt{1+t^2}\, dt$$

$$u = 1+t^2$$
$$du = 2t\, dt$$
$$0.03\,du = 0.06t\, dt$$

$$= \int \sqrt{u}\; 0.03\, du = 0.03\frac{u^{3/2}}{3/2} + C$$
$$= 0.02\,u^{3/2} + C$$
$$= 0.02\,(1+t^2)^{3/2} + C$$

$$Q = 0.02\,(1+t^2)^{3/2} + C$$

USE initial conditions

$t=0 \quad Q = 0.015\,C$

$$0.015 = 0.02\,(1)^{3/2} + C$$
$$C = -0.005$$

The charge is given by $\quad Q = 0.02\,(1+t^2)^{3/2} - 0.005$

At $t = 0.25\,s$ we find Q

$$Q = 0.02\,(1+(0.25)^2)^{3/2} - 0.005$$
$$= \boxed{0.017\,C}$$

#21 $C = 2.5\,\mu F \quad$ At $t=0 \quad V_c = 0$

$i = 25\,mA$

$V = \dfrac{Q}{C}$ $\quad$ but $\quad Q = \int i\, dt$

$$V = \frac{1}{C}\int i\,dt$$

$$= \frac{1}{2.5\mu}\int 25m\,dt$$

$$= \frac{1}{2.5\mu}\,25m\,t + C$$

At $t=0$ $V=0$ therefore $C=0$

$$V = \frac{25m}{2.5\mu}\,t$$

At $t = 12ms$

$$V = \frac{(25m)(12m)}{2.5\mu} = \boxed{120\,V}$$

#23 At $t=0$ $V_c = 4.50mV$ $i = \sqrt[3]{1+6t}$ $C = 3.75\,\mu F$

$$V_c = \frac{1}{C}\int i\,dt$$

$$= \frac{1}{3.75\mu}\int \sqrt[3]{1+6t}\,\mu\,dt$$

INTEGRAL

$$\int (1+6t)^{1/3}\,dt \qquad U = 1+6t \quad du = 6\,dt$$

$$= \frac{1}{6}\int U^{1/3}\,du$$

$$= \frac{1}{6}\,\frac{U^{4/3}}{4/3} + C$$

$$= \frac{1}{8}U^{4/3} + C = \frac{1}{8}(1+6t)^{4/3} + C$$

$$V_c = \frac{1}{3.75\mu}\left(\frac{1}{8}(1+6t)^{4/3}\mu\right) + C$$

At $t=0$ $V_c = 4.50mV$

$$4.5m = \frac{1}{3.75}\left(\frac{1}{8}\right) + C$$

$$C = -0.02883$$

so $V_c = \frac{1}{30}(1+6t)^{4/3} - 0.02883\ mV$

AT $t = 0.565ms$

$$\boxed{V_c = 4.65\,mV}$$

(6 marks)