

CALCULUS I FOR ELECTRONICS ENGINEERING TECHNOLOGY
GRAPHING FUNCTIONS - EXERCISES

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(SOLUTIONS)

1- Sketch the function $f(x) = x^4 - 4x^3$

• DOMAIN

\mathbb{R}

• LIMITS AT INFINITY

$$\lim_{x \rightarrow \infty} x^3(x-4) \rightarrow \infty$$

$$\lim_{x \rightarrow -\infty} x^3(x-4) \rightarrow \infty$$

• Y & X - INTERCEPTS

WHEN $x=0$ $y=0$

WHEN $y=0$ $0 = x^3(x-4)$ $x=0$ or $x=4$

INTERCEPTS are $(0,0)$ & $(4,0)$

• RELATIVE MAXIMUM & MINIMUMS

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$$

CRITICAL pts $x=0$ & $x=3$

intervals	$(-\infty, 0)$	$(0, 3)$	$(3, \infty)$	
test point	-1	2	4	
sign of f'	-	-	+	
behaviour of $f(x)$	↘	↘	↗	

coordinates of relative maximums and minimums:

RELATIVE MINIMUM AT $x=3$ $(3, -27)$
 $y = 3^4 - 4(3)^3 = -27$

• CONCAVITY & INFLECTION POINTS

$$f''(x) = 12x^2 - 24x = 12x(x-2)$$

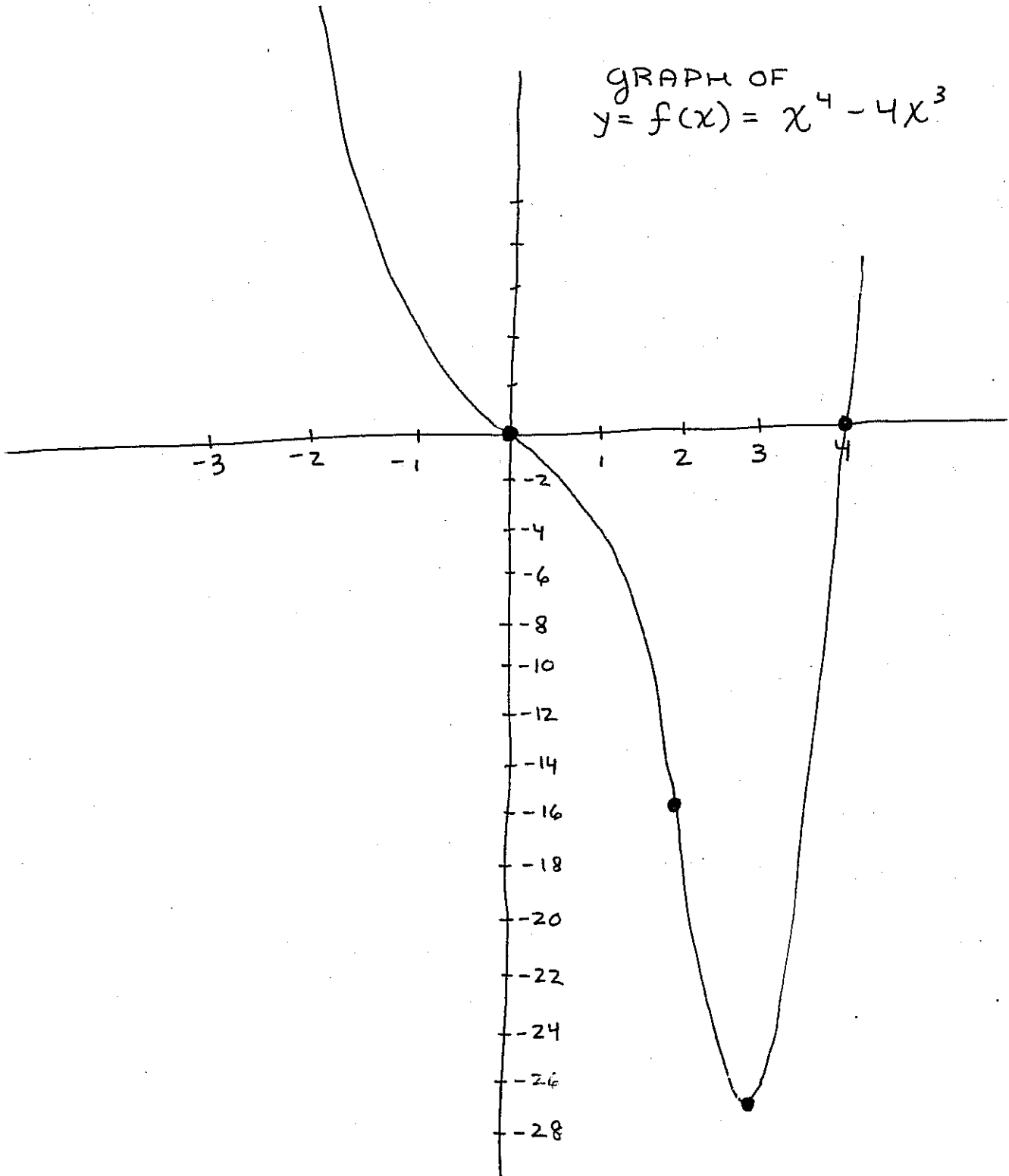
CRITICAL pts AT $x=0$ & $x=2$

intervals	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$	
test point	-1	1	3	
sign of f''	+	-	+	
concavity of $f(x)$	U	∩	U	

coordinates of inflection points:

INFLECTION pts AT $x=0$ $y=0$ $(0,0)$
 AT $x=2$ $y=-16$ $(2,-16)$

GRAPH OF
 $y = f(x) = x^4 - 4x^3$



3 - Sketch the function $f(x) = \frac{x^4 + 1}{x^2}$

• DOMAIN

$$\mathbb{R} \setminus \{0\}$$

• LIMITS AT INFINITY

$$\lim_{x \rightarrow \infty} \frac{x^4/x^2 + 1/x^2}{x^2/x^2} \rightarrow \infty \quad (\text{SAME RESULT FOR } x \rightarrow -\infty)$$

• VERTICAL ASYMPTOTES

$$\text{V.A. at } x=0$$

• Y & X - INTERCEPTS

$$x=0 \quad y \text{ DNE (NO } y \text{ intercept)}$$

$$y=0 \quad x^4 = -1 \text{ IMPOSSIBLE (NO } x \text{-intercept)}$$

• RELATIVE MAXIMUM & MINIMUMS

$$f'(x) = \frac{4x^3(x^2) - 2x(x^4+1)}{x^4} = \frac{4x^5 - 2x^5 - 2x}{x^4} = \frac{2x^4 - 2}{x^3} = \frac{2(x^4 - 1)}{x^3}$$

intervals	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
test point	-2	-0.5	0.5	2
sign of f'	-	+	-	+
behaviour of $f(x)$	\searrow	\nearrow	\searrow	\nearrow

coordinates of relative maximums and minimums:

$$\text{Min AT } x = -1 \quad y = 2 \quad (-1, 2)$$

$$\text{Min AT } x = 1 \quad y = 2 \quad (1, 2)$$

• CONCAVITY & INFLECTION POINTS

$$f''(x) = \frac{8x^3(x^3) - 3x^2(2x^4-2)}{x^6} = \frac{2x^6 + 6x^2}{x^6} = \frac{2(x^4 + 3)}{x^4}$$

intervals	$(-\infty, 0)$	$(0, \infty)$		
test point	-1	1		
sign of f''	+	+		
concavity of $f(x)$	U	U		

coordinates of inflection points:

NO INFLECTION PTS

