Solutions
OPTIMIZATION (ELECTRONICS)

1. $q = 10t^2 - 20t$

$q' = 20t - 20$

$= 20(t-1)$ $\quad t = 1$ critical pt

INTERVALS	$(0,1)$	$(1,\infty)$
test pt	0.5	2
sign of q'	$-$	$+$
BEHAVIOUR OF q	↘	↗

MINIMUM at $t = 1$ sec

MINIMUM charge $q = 10(1)^2 - 20(1)$

$= -10\ C$

MINIMUM OF $q = -10\ C$ at $t = 1$ sec

2. $q = t^2 - 4t + 3$

$q' = 2t - 4$

$= 2(t-2)$

Critical pt $t = 2$

Intervals	$(0,2)$	$(2,\infty)$
TEST pt	1	3
SIGN OF q'	$-$	$+$
BEHAVIOUR OF q	↘	↗

MINIMUM at $t = 2$ sec

$q = 2^2 - 4(2) + 3 = -1\ C$

MINIMUM OF $q = -1\ C$ at $t = 2$ sec

3. $q = Cv$

$q = 0.0000001(0.25t^2 - 2t + 5)$

$q' = 0.00000005t - 0.0000002$

critical pt at $t = 4$

intervals	$(0,4)$	$(4,\infty)$
test pt	1	5
sign of q'	$-$	$+$
behaviour of q	↘	↗

min at $t = 4s$

$q = 0.0000001(0.25(4)^2 - 2(4) + 5)$

$= 0.1\ \mu C$

MINIMUM CHARGE OF $0.1\mu C$ at $t = 4s$

4. $q = Cv$

$= 2(-t^2 + 4t + 68)$

$q' = -4t + 8$

$= -4(t-2)$

$t = 2$ sec

intervals	$(0,2)$	$(2,\infty)$
test pt	1	3
sign of q'	$+$	$-$
behaviour of q	↗	↘

max at $t = 2s$

$q = 2(-(2)^2 + 4(2) + 68)$

$= 144\ \mu C$

MAXIMUM CHARGE OF $144\mu C$ at $t = 2s$

5. OHM'S LAW for AN inductor

$$V = L\frac{di}{dt} \qquad L = 0.5\ H$$

$$i = -t^3/24 + 5t^2/8 + 9t \ \text{mA}$$

$$\frac{di}{dt} = -t^2/8 + 5t/4 + 9$$

$$V = 0.5t^2/8 + 2.5t/4 + 4.5 \ \text{mV}$$

$$V' = -\frac{1t}{8} + \frac{2.5}{4}$$

critical pt

$$0 = -\frac{1t}{8} + \frac{2.5}{4}$$

$$-\frac{2.5}{4} = -\frac{1t}{8}$$

$$t = 5 \text{ sec}$$

Intervals	(0, 5)	(5, ∞)
test pt	1	6
sign of V'	+	—
BEHAVIOUR	↗	↘

MAX At $t=5$

$$V = -0.5(5^2/8) + 2.5(5/4) + 4.5 \ \text{mV}$$

$$= 6.0625 \ \text{mV}$$

$$i = -5^3/24 + 5(5)^2/8 + 9(5) \ \text{mA}$$

$$= 55.42 \ \text{mA}$$

MAXimum $V = 6.0625$ mV at $t = 5$ s
& $i = 55.42$ mA

6. $i = -t^2/8 + t + 2 \text{ mA} \qquad L = 0.5 \text{ H}$

$$v = L \frac{di}{dt} \text{ mV}$$

$$v = 0.5\left(-2t/8 + 1\right) \text{ mV}$$

$$v = -\frac{t}{8} + \frac{1}{2} \qquad v' = -1/8$$

critical pts :

NONE : voltage is decreasing

MAX at $t = 0 \qquad v = 0.5 \text{ mV} \qquad i = 2 \text{ mA}$

7.

$$|Z| = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}$$

derive with respect to ω

$$|Z|' = \frac{1}{2}\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{-1/2} 2\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{C\omega^2}\right)$$

$$0 = \frac{\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{C\omega^2}\right)}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$0 = \left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{C\omega^2}\right)$$

$$0 = \left(\frac{\omega^3 CL}{\omega^2 C} - \frac{\omega}{\omega^2 C}\right)\left(\frac{\omega^2 CL + 1}{\omega^2 C}\right)$$

$$= \left(\omega^3 CL - \omega\right)\left(\omega^2 CL + 1\right)$$

$$= \omega\left(\omega^2 CL - 1\right)\left(\omega^2 CL + 1\right)$$

$$= \omega\left(\omega^4 C^2 L^2 - 1\right)$$

$$\omega^4 C^2 L^2 - 1 = 0$$

$$\omega^4 = \frac{1}{C^2 L^2} \qquad \omega = \sqrt[4]{\frac{1}{C^2 L^2}} = \boxed{\frac{1}{\sqrt{CL}}}$$

8.

$$A = \frac{50\,R_L}{2500 + R_L}$$

$$A' = \frac{50(2500 + R_L) - 50R_L}{(2500+R_L)^2}$$

$$= \frac{125000}{(2500+R_L)^2}$$

A' is always positive there are no critical pts
THIS MEANS A is always increasing

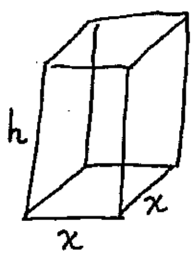
WHAT HAPPENS AS $R_L \to \infty$?

$$\lim_{R_L\to\infty} \frac{50R_L}{2500+R_L} = \lim_{R_L\to\infty} \frac{50R_L/R_L}{2500/R_L + R_L/R_L} = \lim_{R_L\to\infty} \frac{50}{\frac{2500}{R_L}+1}$$

$$= 50$$

THE MAX value of A tends to 50

9.



$V = 650 \text{ in}^3$

$x^2 h = 650$

$h = \frac{650}{x^2}$

MINIMIZE SURFACE AREA

$$A = 4xh + 2x^2$$

$$= 4x\left(\frac{650}{x^2}\right) + 2x^2$$

$$= \frac{2600}{x} + 2x^2$$

$$A' = -\frac{2600}{x^2} + 4x$$

Critical pts: $x = 0$ (A' DNE)

$(0 = A')$ $\quad 0 = -\frac{2600}{x^2} + 4x$

$2600 = 4x^3$

$x^3 = 650$

$x = 8.66$ in

INTERVALS	(0, 8.66)	(8.66, ∞)
test pt	1	9
Sign of A'	—	+
BEHAVIOUR of A	↘	↗

minimum at $x = 8.66$ in
$h = 8.66$ in

10. $i = \frac{t^3}{3} - t$

$i' = \frac{3t^2}{3} - 1$

$= t^2 - 1$

critical pts at $t = \pm 1$

intervals	(0, 1)	(1, ∞)
test pt	0.5	2
Sign of i'	—	+
BEHAVIOUR of i	↘	↗

minimum at $t = 1$ sec
minimum current $i = -2/3$ A

Power $= i^2 R$

$P = \left(\frac{t^3}{3} - t\right)^2 R$

$P = R\left(\frac{t^6}{9} - \frac{2t^4}{3} + t^2\right)$

$P' = R\left(\frac{2}{3}t^5 - \frac{8}{3}t^3 + 2t\right)$

$= \frac{2}{3}Rt\left(t^4 - 4t^2 + 3\right)$

$= \frac{2}{3}Rt\,(t^2 - 3)(t^2 - 1)$

critical points $t = \pm\sqrt{3}, \pm 1, 0$

INTERVALS	(0, 1)	(1, √3)	(√3, ∞)
test pt	0.5	1.5	2
Sign of P'	+	—	+
BEHAVIOUR of P	↗	↘	↗

MAX at $t = 1$ sec

11. $q = 4t^4 - t^3$

$i = dq/dt$

$i = 16t^3 - 3t^2$

$i' = 48t^2 - 6t$
$= 6t(8t - 1)$ critical pts $t = 0$, $t = 1/8$

intervals	$(0, 1/8)$	$(1/8, \infty)$
test pt	0.1	1
sign of i'	—	+
behaviour of i	↘	↗

minimum $t = 1/8$ s

$i = -\frac{1}{64}$ A

max current is $1/64$ A (in negative sense)

12. $V = L\frac{di}{dt}$

$i = 16t^3 - 3t^2$

$\frac{di}{dt} = 48t^2 - 6t$

$V = 16(48t^2 - 6t)$

$V' = 16(96t - 6)$ critical pt $t = 0.0625$ s

intervals	$(0, 0.0625)$	$(0.0625, \infty)$
test pt.	0.01	1
sign V'	—	+
behaviour V	↘	↗

There is no max at $t = 0.0625$

$V = 16(48(0.0625)^2 - 6(0.0625))$
$= -3$ V

13. $$Z = \left[R^2 + (X_L - X_c)^2\right]^{1/2}$$

$$= \left[625\,000 + (1500 - X_c)^2\right]^{1/2}$$

$$Z' = \frac{1}{2}\left[625\,000 + (1500 - X_c)^2\right]^{-1/2} 2(1500 - X_c)(-1)$$

$$= \frac{X_c - 1500}{\sqrt{625\,000 + (1500 - X_c)^2}}$$

CRITICAL POINT AT $X_c = 1500$

INTERVALS	$(-\infty, 1500)$	$(1500, \infty)$
TEST pt	1	1600
sign of Z'	$-$	$+$
BEHAVIOUR of Z	↘	↗

Z is A minimum at $X_c = 1500$

minimum value of $Z = \sqrt{625\,000}\ \Omega$

14. $$R_1 + R_2 = 32$$
$$R_2 = 32 - R_1$$

IN PARALLEL
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2}$$
$$R = \frac{R_1 R_2}{R_2 + R_1}$$
$$R = \frac{R_1(32 - R_1)}{32}$$
$$R = R_1 - \frac{1}{32} R_1^2$$
$$R' = 1 - \frac{1}{16} R_1$$

CRITICAL pt AT $R_1 = 16\Omega$

INTERVALS	$(-\infty, 16)$	$(16, \infty)$
test pt	1	17
sign of R'	+	—
BEHAVIOUR OF R	↗	↘

MAX AT $R_1 = 16\Omega$, $R_2 = 16\Omega$

15. $P = \dfrac{144r}{(r+0.6)^2}$

$$P' = \frac{144(r+0.6)^2 - 2(r+0.6)(144r)}{(r+0.6)}$$

$$= \frac{(r+0.6)(144)\left(r+0.6-2r\right)}{(r+0.6)^4}$$

$$= \frac{144(-r+0.6)}{(r+0.6)^3} \quad \text{critical pt at } r=0.6$$

Intervals	$(-\infty, 0.6)$	$(0.6, \infty)$
Sign of P'	+	—
test pt	0.1	1
Behaviour of P	↗	↘

MAX WHEN $r=0.6$
& WHEN $r=-0.6$

16. $q = Cv$

$q = 8\,(250t^2 - 200t^3)$

$i = \dfrac{dq}{dt} \qquad i = 8\,(500t - 600t^2)$

Maximizing i: $i' = 8\,(500 - 1200t)$

critical pt $t = 5/12$

Intervals	$(0, 5/12)$	$(5/12, \infty)$
test pt.	0.1	1
Sign of i'	+	—
Behaviour of i	↗	↘

MAX At $t = 5/12$ s

$$i = 8\left(500(5/12) - 600(5/12)^2\right) = \boxed{8.33 \times 10^{-4}\,A}$$

Optimization (Electronics)

Calculus I for Electronics Engineering Technology

Winter 2011

Instructor Emilie Richer

1- The charge flowing in an electronic device is given by $q = 10t^2 - 20t$. At what value(s) of t will q be a minimum or maximum? State whether q is maximum or minimum. What is the value of q at that time?

2- The charge passing through a semiconductor junction is given by $q = t^2 - 4t + 3$. What is the value of q and t when q is a maximum or minimum? State whether q is a maximum or minimum.

3- The charging voltage for a capacitor is given by $v = 0.25t^2 - 2t + 5$ V. Determine the amount of maximum or minimum charge and the time at which it occurs. C = 0.1 μF ($q = Cv$)

4- If a capacitor C = 2 μF is being charged by $v = -t^2 + 4t + 68$ V, determine q and t when q is a maximum or minimum.

5- A current $i = -t^3/24 + 5t^2/8 + 9t$ mA is flowing through an inductance L = 500 mH. Determine i, $|v_{Ind}|$, and t for maximum/minimum $|v_{Ind}|$.

6- Repeat exercise 5- for a current $i = -t^2/8 + t + 2$ mA.

7- In a series RLC circuit excited by a sinusoidal current whose angular velocity is ω, the magnitude of the impedance is given by:
$|Z| = [R^2 + (\omega L - 1/(\omega C))^2]^{½}$
As ω is varied, a phenomenon called resonance occurs when $|Z|$ is a minimum. Determine the expression for ω at resonance.

8- The gain of a certain electronic amplifier is given by $A = 50R_L/(2500 + R_L)$. For what value of R_L is A a maximum? What is the gain at this value of R_L?

9- A rectangular apparatus box it to have a square bottom and is to contain 650 in^3. Find its dimensions so as to use the least metal for the box, including the cover.

10- During a certain interval the current in a resistor of R ohms is $i = t^3/3 - t$ A. When does the minimum current flow in the resistor? After what time is the power in the resistor a maximum?

11- The charge transmitted through a circuit varied according to $q = 4t^4 - t^3$ coulombs. After what time t in seconds did the current reach a maximum in the negative sense?

12- If the circuit of problem 11- included a16-henry series inductor, what would have been the greatest *positive* voltage induced in the inductor?

13- The impedance Z (in Ω) in an electric circuit is given by $Z = [R^2 + (X_L - X_C)^2]^{½}$. If R = 2500Ω and X_L = 1500Ω, what value of X_C makes the impedance a minimum?

14- When two electric resistors R_1 and R_2 are in series, their total resistance is 32Ω. If the same resistors are in parallel their total resistance is the maximum possible for two such resistors. What is the resistance of each?

15- The electric power P (in W) produced by a certain battery is given by $P = 144r/(r + 0.6)^2$, where r is the resistance in the circuit. For what value of r is the power a maximum?

16- Find the greatest current in an 8μF capacitor if the applied voltage is $v = 250t^2 - 200t^3$ volts.