NAME: _____ *Solutions* _____

QUIZ 1 - Graphs & Functions
Calculus for Electronics Engineering Technology
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Graphs of Linear Functions

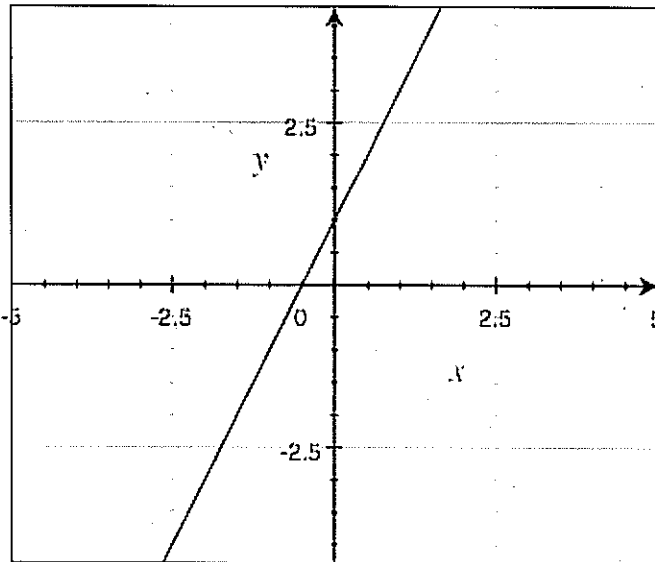State the general form of a linear function:

$$y = mx + b$$

2- How do you find the slope "m" of a line?

You need two points (x_1, y_1) & (x_2, y_2),

$$m = (y_2 - y_1) / (x_2 - x_1)$$

3- Find the equation of a line passing through the points (-2 , 3) and (1 , 9)

$$m = \frac{9-3}{1-(-2)} = \frac{6}{3} = 2 \quad \text{so} \quad y = 2x+b : \quad 3 = 2(-2)+b : b=7 : \boxed{y = 2x+7}$$

4- Give the equation of the line pictured below

TWO POINTS ARE
$(-0.5, 0)$ & $(0, 1)$

$$m = \frac{1-0}{0-(-0.5)} = 2$$

$$y = 2x + b$$
$$1 = 2(0) + b$$
$$b = 1$$

$$\boxed{y = 2x + 1}$$

5- Give the equation of the line perpendicular to the one pictured above and passing through the point (-0.5 , 0)

PERPENDICULAR LINE WILL HAVE NEGATIVE RECIPROCAL AS A SLOPE, that is $m = -\frac{1}{2}$

$$y = -\frac{1}{2}x + b$$
$$0 = -\frac{1}{2}(-0.5) + b$$
$$b = -0.25$$

$$\boxed{y = -0.5x - 0.25}$$

Graphs of Quadratic Functions

State the general form of a quadratic function:

$$y = ax^2 + bx + c$$

2- What determines whether the graph is concave upwards or downwards (explain):

IF $a > 0$ the graph is concave upwards

IF $a < 0$ the graph is concave downwards

3- What is the formula for the x-coordinate of the vertex of quadratic function?

$$x = -b/2a$$

4- Once you have found the x-coordinate of the vertex, how do you find the y-coordinate?

plug the x-value into the formula

$$y = ax^2 + bx + c$$

5- The following questions pertain to the graph pictured below.

a- For what values of x does y=0?
$$x = 2, 8$$

b- What is the vertex? $(5, -9)$

c- Is the quadratic concave upwards or concave downwards?

upwards

d- State three points on the graph.
$(2,0), (8,0), (5,-9)$

(BONUS) Give the equation of the quadratic function corresponding to the graph?

$$y = ax^2 + bx + c$$

plug in three points to get 3 equations
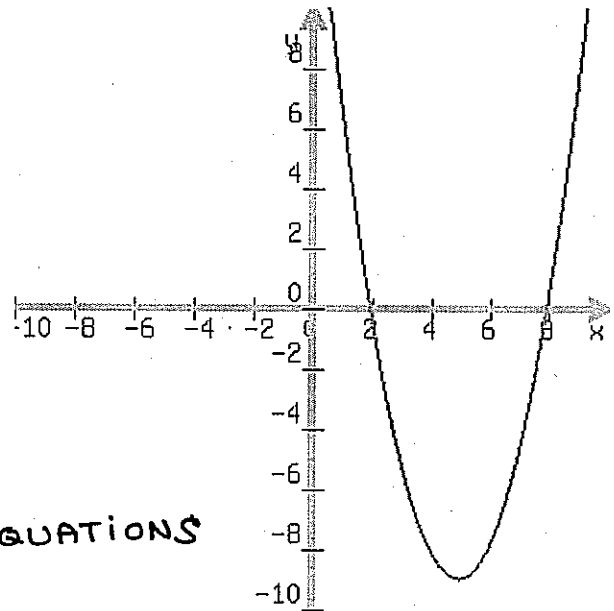
$$0 = a(2)^2 + b(2) + c$$
① $$0 = 4a + 2b + c$$

$$0 = a(8)^2 + b(8) + c$$
② $$0 = 64a + 8b + c$$

③ $$-9 = a(5)^2 + b(5) + c$$
$$-9 = 25a + 5b + c$$

THIS SYSTEM OF 3 EQUATIONS & 3 UNKNOWNS YIELDS $a = 1$
$$b = -10$$
$$c = 16$$

THE FUNCTION HAS EQUATION

$$y = x^2 - 10x + 16$$

Graphs of Sinusoidal Functions

What is the period of the graph y = sinx?   2π

2- How many times does the graph y = sinx cross the y-axis?

   ONCE AT THE POINT (0,0)

3- How many times does the graph y = sinx cross the x-axis?

   ∞ MANY TIMES
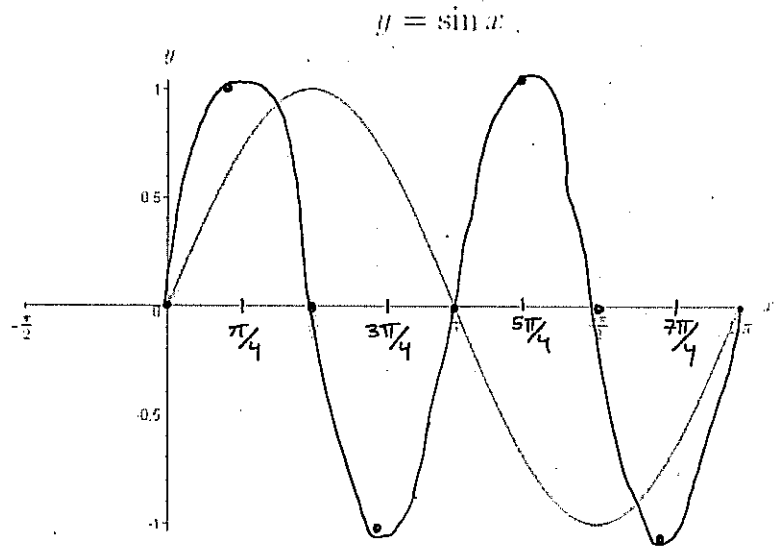
4- Where does the graph of y = sinx cross the x-axis?

   ALL MULTIPLES of π
   $x = ... -2π, -π, 0, π, 2π, 3π ...$

5- Graph the function y = sin(2x)

TABLE

ANGLE (2x)	x	y = SIN 2x
0	0	0
π/2	π/4	1
π	π/2	0
3π/2	3π/4	−1
2π	π	0

$y = \sin x$

6- What is the period of the graph y = cosx?   2π

7- How many times does the graph y = cosx cross the y-axis?

ONCE   AT   (0,1)

8- How many times does the graph y = cosx cross the x-axis?

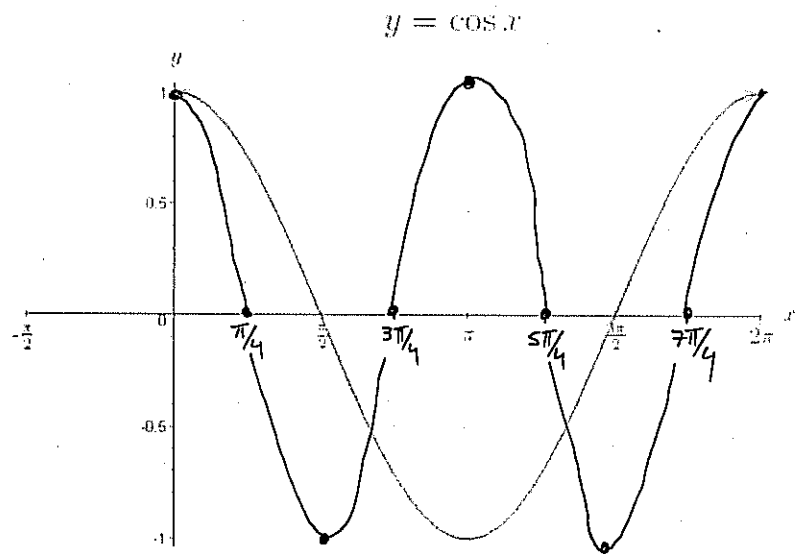∞ MANY TIMES

9- Where does the graph of y = cosx cross the x-axis?

$\frac{\pi}{2} \pm$ MULTIPLE OF π

10- Graph the function y = cos(2x)

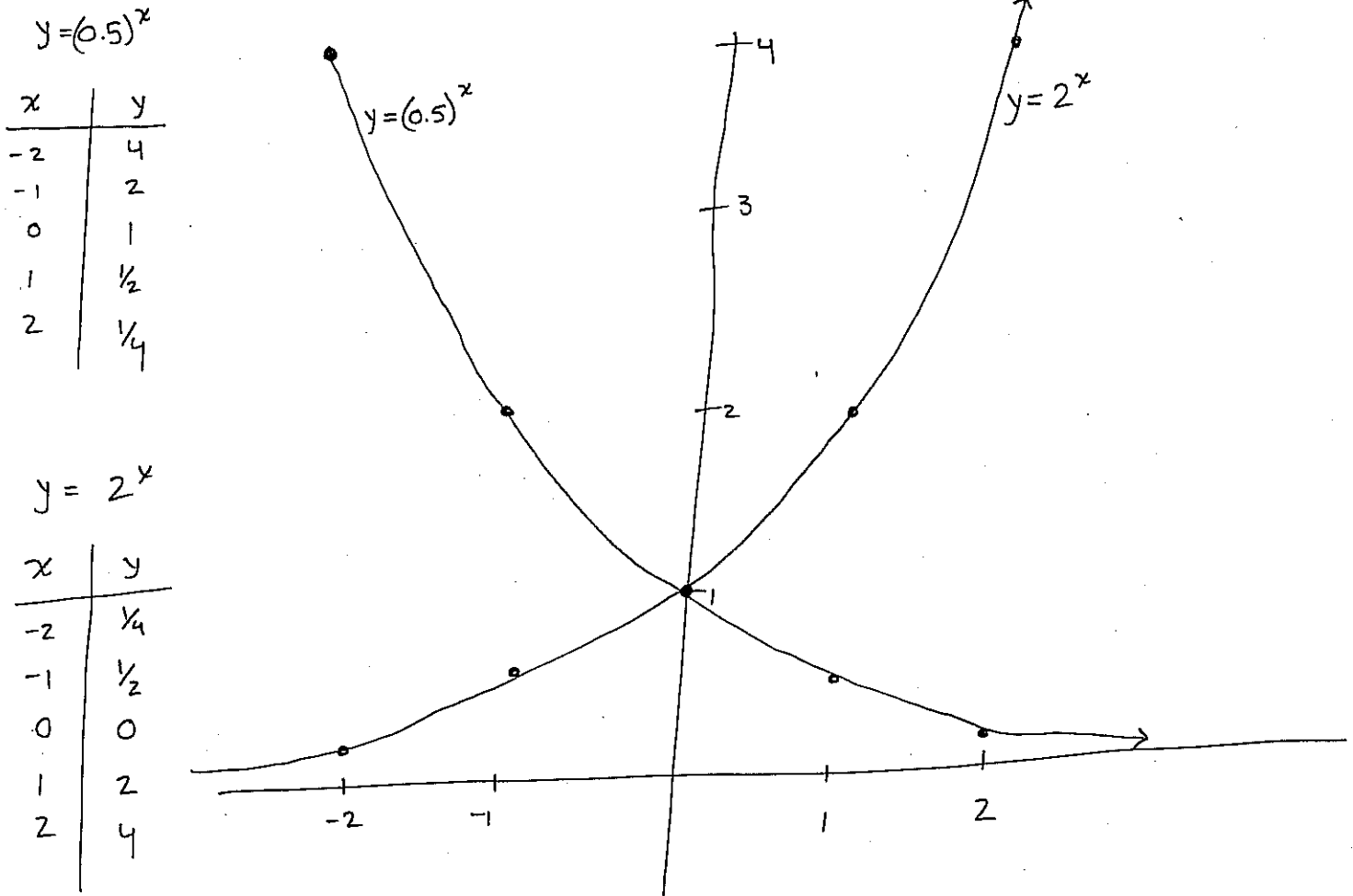| ANGLE 2x	x	y = cos2x
0	0	1
$\frac{\pi}{2}$	$\frac{\pi}{4}$	0
π	$\frac{\pi}{2}$	−1
$\frac{3\pi}{2}$	$\frac{3\pi}{4}$	0
2π	π	1

$y = \cos x$

Graphs of Exponential Functions

Exponential functions have the form $y = A(b^x)$ (A, b real numbers) $b > 0$ $b \neq 1$

1- Sketch the function $y = (0.5)^x$ and the graph $y = 2^x$ on the same graph. Build a table of values with at least 5 points for each of these graphs.

$y = (0.5)^x$

x	y
-2	4
-1	2
0	1
1	½
2	¼

$y = 2^x$

x	y
-2	¼
-1	½
0	0
1	2
2	4

2- The function $y = (0.5)^x$ is called an **exponential decay** function and the function is called an **exponential growth** function. Can think of an explanation for these names?

decay : MAGNITUDE OF y GETS SMALLER AS x increases

growth: THE MAGnitude of y gets larger AS x increases

3- What part of the function $y = A(b^x)$ determines whether the exponential functional is a decay function or whether it is an growth function?

THE base 'b'

if   b < 1   decay

if   b > 1   growth

Graphs of Logarithmic Functions

1- Give the exponential form of the following logarithmic functions

a- $y = \log_2 x$ $2^y = x$

b- $y = -2\ln x$ $\dfrac{y}{-2} = \ln x \implies e^{y/-2} = x$

c- $y = \log_3 (2x)$ $3^y = 2x \implies \left(\dfrac{1}{2}\right)(3^y) = x$

d- $y = \log_4(x+1)$ $4^y = x+1 \implies x = 4^y - 1$

e- $y = \log_2(x-1) + 2$

$$y - 2 = \log_2(x-1)$$
$$2^{y-2} = x - 1$$
$$x = 2^{y-2} + 1$$

2- Graph the function $y = \log_2 (x - 1)$ using its exponential form and a table of values with at least 5 points.

$$2^y = x - 1$$
$$x = 2^y + 1$$

y	x
-2	$5/4$
-1	$3/2$
0	2
1	3
2	5