NAME: SOLUTIONS

QUIZ 3

Dawson College
Course Code: 201-NYA-05 S07
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Question 1. *(5 marks)*
Using limits, find the slope of the tangent to the curve $f(x) = \sqrt{x-1}$ at the point $(2,1)$.

$$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(2+\Delta x) - f(2)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{2+\Delta x - 1} - 1}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{1+\Delta x} - 1}{\Delta x} \left(\frac{\sqrt{1+\Delta x}+1}{\sqrt{1+\Delta x}+1}\right)$$

$$= \lim_{\Delta x \to 0} \frac{1+\Delta x - 1}{\Delta x\,(\sqrt{1+\Delta x}+1)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{1+\Delta x}+1}$$

$$= \boxed{\frac{1}{2}}$$

Question 2. *(5 marks)*
Using limits, find the derivative of $f(x) = x^2 - 3x$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[(x+\Delta x)^2 - 3(x+\Delta x)] - [x^2 - 3x]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 3x - 3\Delta x - x^2 + 3x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x\,(2x + \Delta x - 3)}{\Delta x}$$

$$= \boxed{2x - 3}$$

Question 3. *(5 marks)*
Determine if the function $g(x)$ is continuous at $x = 2$. **Justify** your answer using the conditions necessary for continuity and **proper notation**.

$$g(x) = \begin{cases} 5 & x < 2 \\ 3x - 2 & x = 2 \\ 5 + 2x - x^2 & x > 2 \end{cases}$$

① $g(2)$ EXISTS $g(2) = 3(2) - 2 = 4$

② $\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} 5 = 5$

$\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} 5 + 2x - x^2 = 5 + 4 - 4 = 5$

$\lim_{x \to 2} g(x)$ EXISTS

③ $g(2) \neq \lim_{x \to 2} g(x)$

SO g IS NOT CONTINUOUS AT $x = 2$

Question 4. *(5 marks)*
Determine for which values of x the function $f(x) = \frac{\sqrt{x+1}}{x-2}$ is **not continuous**. **Justify** your answer using the conditions necessary for continuity and **proper notation**.

① f does not exist for $x < -1$ & for $x = 2$
So it is NOT CONTINUOUS there

② $\lim_{x \to -1^-} f(x)$ does not exist so f is NOT CONTINUOUS for $x = -1$