NAME: SOLUTIONS

STUDENT ID NUMBER: ____________________

TEST 1

DAWSON COLLEGE

Calculus 1 (Electronics Engineering Technology)

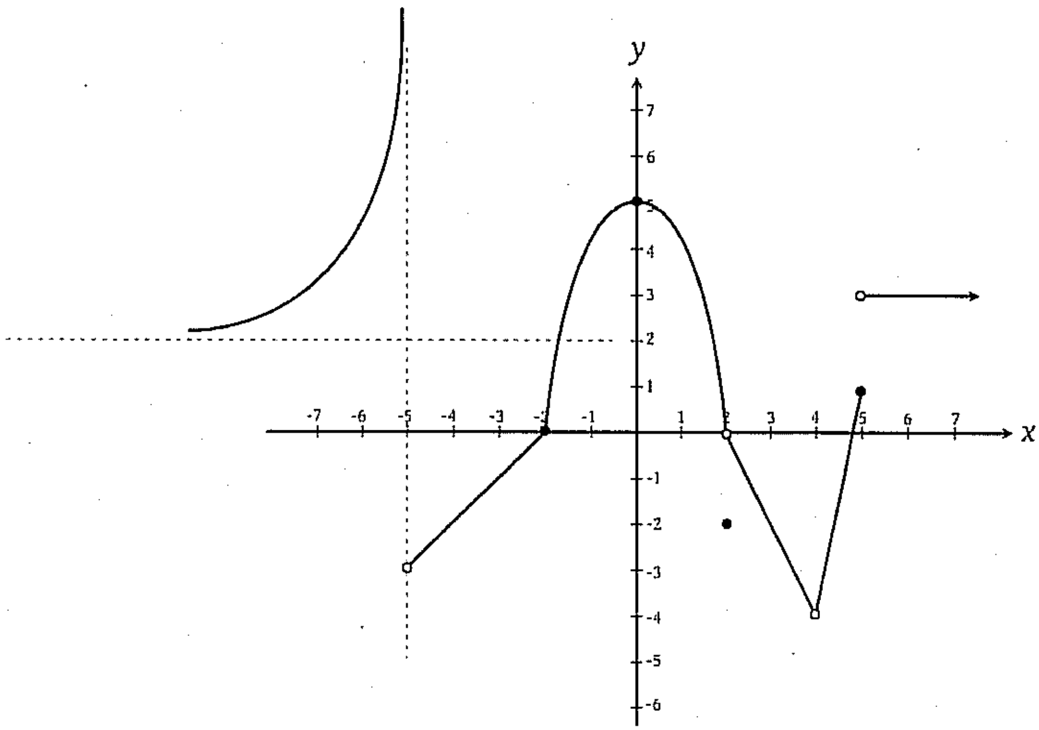
201-NYA-05

Instructor: Emilie Richer

Date: Feb. 25th 2011

THIS TEST IS MARKED OUT OF **50 points**

Question 1. *(6 marks)*



(a) Determine for which values of x the function $y = f(x)$ pictured above is **not continuous**.
(b) For each of the x values found in (a) give the reason why the function is not continuous using the three conditions required for continuity.
Note that you must use proper notation and terminology in your justification

(a) $f(x)$ is NOT CONTINUOUS FOR $x=-5$, $x=2$, $x=4$, $x=5$

(b) $f(x)$ is NOT CONTINUOUS AT

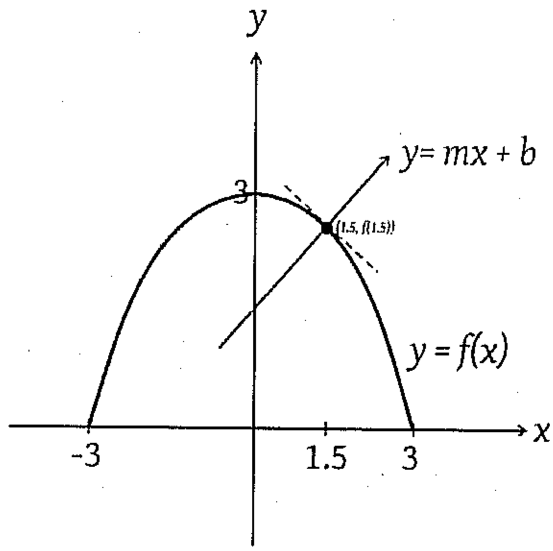
$x=-5$ because $f(-5)$ DNE ($\lim_{x\to -5} f(x)$ DNE either)

$x=2$ because $f(2) \neq \lim_{x\to 2} f(x)$

$x=4$ because $f(4)$ DNE

$x=5$ because $\lim_{x\to 5} f(x)$ DNE

Question 2. *(5 marks)*



DISCLAIMER: THE graph pictured here is the graph OF $y = 3\cos\frac{\pi}{6}x$ SORRY :(

Find the equation of the line $y = mx + b$ pictured above. This line is perpendicular to the tangent line to the function $y = f(x) = \cos(\frac{\pi}{6}x)$ at $x = 1.5$

- Finding $f'(x)$ will give the slope of the tangent

  $f'(x) = -\sin(\pi/6\, x) \cdot \pi/6$

  (AT 1.5) $= -\sin(\pi/4) \cdot \pi/6 = -\frac{1}{\sqrt{2}} \cdot \frac{\pi}{6}$ (-0.370)

- Slope of perpendicular line is negative reciprocal

  $m = \frac{6\sqrt{2}}{\pi}$ (2.7)

- Point on the line: $(1.5, \cos(\frac{\pi}{6}(1.5))) = (1.5, 1/\sqrt{2})$

- $y = mx + b$

  $\frac{1}{\sqrt{2}} = \frac{9\sqrt{2}}{\pi} + b \longrightarrow b = \frac{1}{\sqrt{2}} - \frac{9\sqrt{2}}{\pi} = \frac{\pi - 18}{\sqrt{2}\pi}$ (-3.34)

- The equation is $\boxed{y = \frac{6\sqrt{2}}{\pi}x + \frac{\pi - 18}{\sqrt{2}\pi}}$ $(y = 2.7x - 3.34)$

Question 3. *(4 marks)*
Use the limit definition of the derivative to find the derivative of the function $f(x) = -2x^2 + 3x$. Note that no marks will be given if you do not use the limit definition to evaluate the derivative.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-2(x+\Delta x)^2 + 3(x+\Delta x) - (-2x^2+3x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-2(x^2 + 2x\Delta x + \Delta x^2) + 3x + 3\Delta x + 2x^2 - 3x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{-2x^2} - 4x\Delta x - 2\Delta x^2 \cancel{+3x} + 3\Delta x \cancel{+2x^2} \cancel{-3x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-4x\Delta x - 2\Delta x^2 + 3\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(-4x - 2\Delta x + 3)}{\cancel{\Delta x}}$$

$$= -4x + 3$$

Question 4. *(1 mark each)*

Find the derivative of each of the following functions.

(a) $f(x) = 3x\sin x$

$f'(x) = \boxed{3\sin x + (\cos x)3x}$

(b) $f(x) = 2\sqrt{x} - \frac{2}{x^4}$ $= 2x^{1/2} - 2x^{-4}$

$f'(x) = x^{-1/2} + 8x^{-5} = \boxed{\frac{1}{\sqrt{x}} + \frac{8}{x^5}}$

(c) $h(t) = \tan(3t)$

$h'(t) = \boxed{\sec^2(3t) \cdot 3}$

(d) $g(t) = e^{\cos t}$

$g'(t) = \boxed{e^{\cos t}(-\sin t)}$

(e) $g(t) = \ln(5x^2 - 7x)$

$g'(x) = \boxed{\frac{1}{5x^2 - 7x}(10x - 7)}$

Question 5. *(3 marks each)*
Find the derivative of each of the following functions.

(a) $f(x) = 3x^2 \cos(\sin x)$

$f'(x) = \boxed{6x\cos(\sin x) + (-\sin(\sin x))\cos x\,(3x^2)}$

(b) $f(x) = \frac{e^{4x}}{\cos(12x^3 - x)}$

$f'(x) = \boxed{\dfrac{e^{4x} \cdot 4\cos(12x^3 - x) + \sin(12x^3 - x)(36x^2 - 1)\,e^{4x}}{\cos(12x^3 - x)}}$

(c) $h(t) = \sqrt{3t\tan(3t)} = (3t \tan 3t)^{1/2}$

$h'(t) = \boxed{\frac{1}{2}(3t\tan 3t)^{-1/2}\left(3\tan 3t + (\sec^2 3t)(3)(3t)\right)}$

(d) $g(t) = \underbrace{4te^{\cos t}}_{\text{FUNCTION 1}}\underbrace{\sin 5t}_{\text{FUNCTION 2}}$

$g'(t) = \boxed{\left(4e^{\cos t} + e^{\cos t}(-\sin t)4t\right)\sin 5t + (\cos 5t)(5)(4te^{\cos t})}$

(e) $g(t) = \ln(\sin^2 x) = \ln((\sin x)^2)$

$g'(t) = \frac{1}{\sin^2 x} 2\sin x \cos x = \frac{2\cos x}{\sin x} = \boxed{2\cot x}$

Question 6. *(2 marks each)*
Evaluate the following limits algebraically (not using a table of values).

(a) $\lim_{x \to 4} \frac{2x^2-9x+4}{16-x^2}$

FACToring: $2x^2 - 9x + 4$
$= 2x^2 - 8x - x + 4$
$= 2x(x-4) - 1(x-4)$
$= (x-4)(2x-1)$

$= \lim_{x \to 4} \frac{(x-4)(2x-1)}{(4-x)(4+x)}$

$= \lim_{x \to 4} \frac{\cancel{(x-4)}(2x-1)}{-\cancel{(x-4)}(4+x)}$

$= \boxed{\frac{7}{-8}}$

(b) $\lim_{x \to \infty} \frac{7x^5-x^2+2}{x^7+1}$

$= \lim_{x \to \infty} \frac{7x^5/x^7 - x^2/x^7 + 2/x^7}{x^7/x^7 + 1/x^7} = \lim_{x \to \infty} \frac{\cancel{7/x^2}^{\to 0} - \cancel{1/x^5}^{\to 0} + \cancel{2/x^7}^{\to 0}}{1 + \cancel{1/x^7}^{\to 0}} = \frac{0}{1} = \boxed{0}$

(c) $\lim_{x \to 25} \frac{x-25}{\sqrt{x}-5}$

$= \lim_{x \to 25} \frac{(x-25)(\sqrt{x}+5)}{(\sqrt{x}-5)(\sqrt{x}+5)}$

$= \lim_{x \to 25} \frac{\cancel{(x-25)}(\sqrt{x}+5)}{\cancel{(x-25)}}$

$= \sqrt{25} + 5 = \boxed{10}$

(d) $\lim_{x \to -\infty} \frac{-3x^3-2x}{x^2+1} = \lim_{x \to -\infty} \frac{-3x^3/x^2 - 2x/x^2}{x^2/x^2 + 1/x^2} = \lim_{x \to -\infty} \frac{-3x - \cancel{2/x}^{\to 0}}{1 + \cancel{1/x^2}^{\to 0}}$

$= \lim_{x \to -\infty} -3x \quad \boxed{\text{DNE}}$

$(\longrightarrow +\infty)$

Question 7. *(3 marks)*
Find the value of the constant a if the slope of the tangent line to the curve $y = 2axe^x$ at $x = 0$ is equal to 3.

y' AT $x=0$ is 3

$y' = 2ae^x + e^x(2ax)$ using product rule

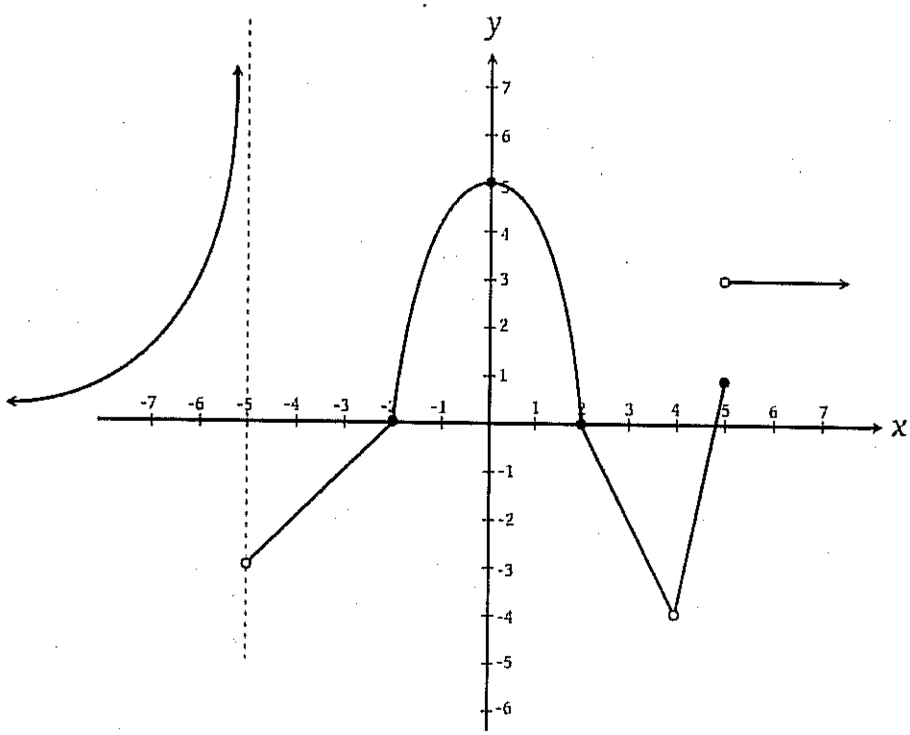
AT $x=0$ $y' = 2ae^0 + e^0(2a)(0)$

$= 2a$

SO $2a = 3$

$$\boxed{a = 3/2}$$

Question 8. *(4 marks)*



Find the following values corresponding to the graph of $f(x)$ pictured above.

(a) $f(5) =$ 1

(b) $\lim_{x \to 0} f(x) =$ 5

(c) $\lim_{x \to -5^+} f(x) =$ -3

(d) $\lim_{x \to 5^-} f(x) =$ 1

(e) $\lim_{x \to 4} f(x) =$ -4

(f) $\lim_{x \to -5^-} f(x) =$ DNE (or $\longrightarrow +\infty$)

(g) $\lim_{x \to -\infty} f(x) =$ 0

(h) $f'(0) =$ 0 (SLOPE OF HORIZONTAL TANGENT LINE)