NAME: ____________________

STUDENT ID NUMBER: SOLUTIONS

TEST 2

DAWSON COLLEGE

Calculus 1 (Electronics Engineering Technology)

201-NYA-05

Instructor: Emilie Richer

Date: April 8th 2011

THIS TEST IS MARKED OUT OF **50 points**

Question 1. *(3 marks)*
Find the **second** derivative of $f(x) = \sqrt[3]{2x-5}$

$$f(x) = (2x-5)^{1/3}$$
$$f'(x) = \frac{1}{3}(2x-5)^{-2/3} \cdot 2$$
$$= \frac{2}{3}(2x-5)^{-2/3}$$
$$= -\frac{4}{9}(2x-5)^{-5/3} \cdot 2$$
$$= \boxed{-\frac{8}{9}(2x-5)^{-5/3}}$$

Question 2. *(5 marks)*
Find the derivative of $y = (\sin x)^{\cos x}$

$$\ln y = \ln(\sin x)^{\cos x}$$
$$\ln y = \cos x \ln(\sin x)$$
$$\frac{1}{y} y' = -\sin x \ln \sin x + \frac{1}{\sin x} \cos x$$
$$\boxed{y' = (\sin x)^{\cos x}\left[-\sin x \ln \sin x + \cot x\right]}$$

Question 3. *(4 marks)*
Determine the intervals on which the function $f(x) = x(x-2)^4$ is increasing.

$f'(x) = (x-2)^4 + 4(x-2)^3 \cdot x$

FACTORING : $= (x-2)^3 (x-2+4x)$

$= (x-2)^3 (5x-2)$

critical pts $x=2$, $x = 2/5$

INTERVALS	$(-\infty, 2/5)$	$(2/5, 2)$	$(2, \infty)$
TEST	0	1	3
Sign of f'	+	−	+
	↗	↘	↗

$f(x)$ is increasing on the intervals $(-\infty, 2/5)$ & $(2, \infty)$

Question 4. *(5 marks)*
Find the derivative y' of $e^{x+2y} + xy^2 = y^3$ by implicit differentiation.

$$e^{x+2y}(1+2y') + y^2 + 2xyy' = 3y^2y'$$

$$e^{x+2y} + 2e^{x+2y}y' + y^2 + 2xyy' = 3y^2y'$$

$$y'(2e^{x+2y} + 2xy - 3y^2) = -y^2 - e^{x+2y}$$

$$\boxed{y' = \frac{-y^2 - e^{x+2y}}{2e^{x+2y} + 2xy - 3y^2}}$$

Question 5. *(5 marks)*
Find the **second** derivative of $f(x) = \tan(3x)$

$$f'(x) = \sec^2(3x) \cdot 3 = \frac{3}{\cos^2(3x)} = 3(\cos 3x)^{-2}$$

$$f''(x) = -6(\cos 3x)^{-3}(-\sin 3x) \cdot 3$$

$$= \boxed{\frac{18 \sin 3x}{(\cos 3x)^3}}$$

Question 6. *(5 marks)*
The charge transmitted through a circuit varies with time t (in seconds) according to $q = -4t^4 + t^3$ coulombs. After what time does the **current** i reach its maximum? What is the value of the maximum current? (Remember: $i = \frac{dq}{dt}$)

$$i = \frac{dq}{dt} = -16t^3 + 3t^2$$

$$i' = -48t^2 + 6t$$
$$= -6t(8t - 1) \quad \text{critical pts } t = 0,\ t = 1/8$$

intervals	$(0, 1/8)$	$(1/8, \infty)$
test	0.1	1
sign i'	+	−
	↗	↘

$\boxed{\begin{array}{l}\text{MAXIMUM current at } t = 1/8 \text{ s} \\ \text{value of max current } \quad i = -16(1/8)^3 + 3(1/8)^2 = 1/64 \text{ A}\end{array}}$

Question 7. *(5 marks)*
The charging voltage of a capacitor of capacitance $C = 0.1\mu$F varies with time t (in seconds) according to $v = 0.25t^2 - 2t + 5$ volts.

(a) Determine if the charge $q = Cv$ reaches a maximum or a minimum.
(b) Determine the time at which the charge reaches its maximum or minimum and determine the value of the charge at this time.

(a) $q = Cv = 0.025t^2 - 0.2t + 0.5$

$q' = 0.05t - 0.2$

critical pt at $t = 4$s

intervals	$(0,4)$	$(4,\infty)$
test	1	5
sign of q'	$-$ ↘	$+$ ↗

$\boxed{\text{minimum charge}}$

(b) min at $t = 4$s

$q = 0.025(4)^2 - 0.2(4) + 0.5 = 0.1\,\mu C$

Question 8. *(4 marks)*
Find the derivative y' of $\cos y = \ln(x+y)$ by implicit differentiation.

$$-\sin y\, y' = \frac{1}{x+y}(1+y')$$

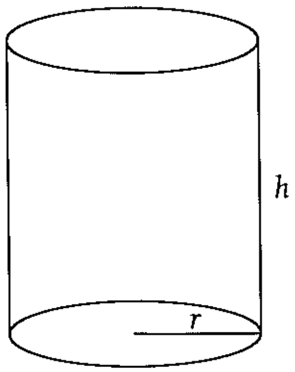
$$-\sin y(x+y)y' = (1+y')$$

$$-\sin y(x+y)y' - y' = 1$$

$$\boxed{y' = \frac{1}{-\sin y(x+y) - 1}}$$

Question 9. *(5 marks)*
A cylindrical container with no top has a surface area of 4π m^2. What height must the container have in order to maximize its volume?

Formulas:
Circumference of a circle with radius $r = 2\pi r$
Volume of a cylinder (with height h and radius of base r) $= \pi r^2 h$



SURFACE AREA = AREA OF BASE + AREA OF SIDE

$$A = \pi r^2 + 2\pi r h$$
$$4\pi = \pi r^2 + 2\pi r h$$
$$h = \frac{4-r^2}{2r}$$

MAXIMIZE VOLUME

$$V = \pi r^2 h$$
$$= \pi r^2 \left(\frac{4-r^2}{2r}\right)$$
$$= \frac{\pi}{2}(4r - r^3)$$

$$V' = \frac{\pi}{2}(4-3r^2)$$ critical pts $4-3r^2=0$, $3r^2=4$, $r^2 = \pm\sqrt{4/3}$

INTERVALS	$(0, \sqrt{4/3})$	$(\sqrt{4/3}, \infty)$
TEST	0.1	2
SIGN OF V'	+	—
	$\nearrow$	$\searrow$

MAX AT $r = \sqrt{4/3}$

WE WANT height:

$$h = \frac{4-r^2}{2r} = \frac{4 - 4/3}{2\sqrt{4/3}} = \frac{\frac{8}{3}}{\frac{4}{\sqrt{3}}} = \frac{8\sqrt{3}}{12} = \boxed{\frac{2\sqrt{3}}{3}}$$

Question 10. *(9 marks)*
Sketch the function $y = x^4 - 2x^2$

Include all the following points in the sketch:
(a) Any x-intercepts and the y-intercept
(b) Any maximums or minimums
(c) Any inflection points

(a) y-INTERCEPT $(0,0)$

x-intercepts
$$0 = x^4 - 2x^2$$
$$= x^2(x^2-2)$$
$$x = 0, \pm\sqrt{2}$$
$$\boxed{(0,0) \quad (-\sqrt{2},0) \quad (\sqrt{2},0)}$$

(b) $y' = 4x^3 - 4x$
$= 4x(x^2-1)$ CRITICAL pts $x=0, \pm 1$

INTERVALS	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
TEST	-2	-0.5	0.5	2
sign y'	$-$	$+$	$-$	$+$
	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$

MAX AT $x=0$ $y=0$ $\boxed{(0,0) \text{ MAX}}$

MINS AT $x=\pm 1$ $y=-1$ $\boxed{(-1,-1) \;\&\; (1,-1) \text{ MINS}}$

(c) $y'' = 12x^2 - 4$
$= 4(3x^2 \quad)$ CRITICAL pts $\pm\sqrt{1/3}$

INTERVALS	$(-\infty, -\sqrt{1/3})$	$(-1/\sqrt{3}, \sqrt{1/3})$	$(1/\sqrt{3}, \infty)$
TEST	-1	0	1
sign y''	$+$	$-$	$+$
CONCAVITY	$\cup$	$\cap$	$\cup$

INFLECTION PTS AT $x = \pm\sqrt{1/3}$

$$y = \left(\sqrt{1/3}\right)^4 - 2\left(\sqrt{1/3}\right)^2$$
$$= \frac{1}{9} - 2\left(\frac{1}{3}\right)$$
$$= \frac{1}{9} - \frac{6}{9} = -\frac{5}{9}$$

$$\boxed{\left(-1/\sqrt{3}, -5/9\right) \quad \& \quad \left(\frac{1}{\sqrt{3}}, -5/9\right)}$$

SKETCH