NAME: SOLUTIONS

STUDENT ID NUMBER: ____________________

TEST 3

DAWSON COLLEGE

Calculus 1 (Electronics Engineering Technology)

201-NYA-05

Instructor: Emilie Richer

Date: May 6th 2011

THIS TEST IS MARKED OUT OF **50 points**

Question 1. *(4 marks each = 20 marks)*
Evaluate the following indefinite integrals.

a. $\int \frac{\arctan^5(3x)}{1+9x^2}\,dx$

$U = \arctan 3x$

$du = \frac{3}{1+9x^2}\,dx$

$\frac{1}{3}du = \frac{1}{1+9x^2}\,dx$

$= \int \frac{1}{3} U^5\,du$

$= \frac{1}{18} U^6 + C$

$= \boxed{\frac{1}{18}\arctan^6(3x)}$

b. $\int \frac{e^{\frac{1}{x}}}{3x^2}\,dx$

$U = 1/x$

$du = -1/x^2\,dx$

$= \int -\frac{1}{3} e^u\,du$

$= -\frac{1}{3} e^u + C$

$= \boxed{-\frac{1}{3} e^{1/x} + C}$

c. $\int \frac{12x^{-3} - 2x^{\frac{1}{4}} + \sqrt{5}x^8 + 5x^3}{x^4}\,dx = \int 12x^{-7} - 2x^{-15/4} + \sqrt{5}x^4 + 5x^{-1}\,dx$

$= 12x^{-6}/{-6} - 2\frac{x^{-11/4}}{-11/4} + \sqrt{5}\,x^5/5 + 5\ln x + C$

$= \boxed{-\frac{2}{x^6} + \frac{8}{11x^{11/4}} + \frac{\sqrt{5}}{5}x^5 + 5\ln x + C}$

d. $\int \frac{\ln^{-6}(x)}{3x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \int \frac{1}{3} u^{-6}\,du$

$= \frac{1}{3}\frac{u^{-5}}{-5} + C$

$= \boxed{-\frac{1}{15}\ln^{-5} x + C}$

e. $\int \frac{8xe^{x^2}}{\sqrt{1-(e^{x^2})^2}}\,dx$

$u = e^{x^2}$

$du = 2xe^{x^2}\,dx$

$= \int \frac{4}{\sqrt{1-u^2}}\,du$

$= 4\arcsin u + C$

$= \boxed{4\arcsin(e^{x^2}) + C}$

Question 2. *(5 marks)*
Give the function $y = f(x)$ that satisfies the following conditions:

1. $f''(x) = \cos(2x) - \frac{1}{x^2} - 2x$
2. The slope of the tangent line of $y = f(x)$ at $x = 1$ is 0
3. The function $f(x)$ passes through the point $(1, -1)$

$$f'(x) = \frac{1}{2}\sin 2x + \frac{1}{x} - x^2 + C$$

$$0 = \frac{1}{2}\sin 2 + 1 - 1 + C$$

$$C = -\frac{1}{2}\sin 2$$

$$f'(x) = \frac{1}{2}\sin 2x + \frac{1}{x} - x^2 - \frac{1}{2}\sin 2$$

$$f(x) = -\frac{1}{4}\cos 2x + \ln x - \frac{1}{3}x^3 - \frac{1}{2}(\sin 2)x + C$$

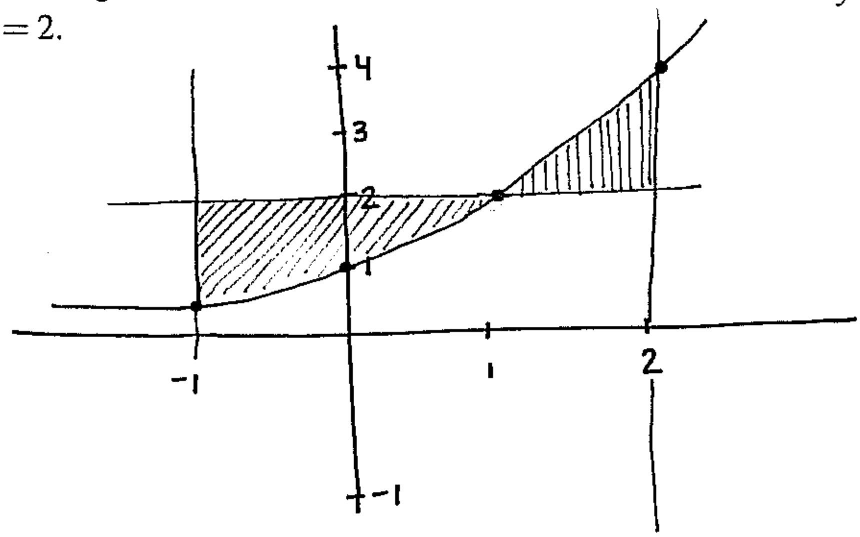
$$-1 = -\frac{1}{4}\cos 2 - \frac{1}{3} - \frac{1}{2}\sin 2 + C$$

$$C = -1 + \frac{1}{4}\cos 2 + \frac{1}{3} + \frac{1}{2}\sin 2$$

$$\approx -0.$$

So $\boxed{f(x) = -\frac{1}{4}\cos 2x + \ln x - \frac{1}{3}x^3 - \frac{1}{2}(\sin 2)x -}$

Question 3. *(5 marks)*
Use integration to find the area enclosed between the curves $y = 2^x$, $y = 2$, $x = -1$, $x = 2$.



$$\text{AREA} = \int_{-1}^{1} 2 - 2^x \, dx + \int_{1}^{2} 2^x - 2 \, dx$$

$$= \left(2x - \frac{1}{\ln 2} 2^x\right)\Big|_{-1}^{1} + \left(\frac{1}{\ln 2} 2^x - 2x\right)\Big|_{1}^{2}$$

$$= \left[\left(2 - \frac{2}{\ln 2}\right) - \left(-2 - \frac{1}{2\ln 2}\right)\right] + \left[\left(\frac{4}{\ln 2} - 4\right) - \left(\frac{2}{\ln 2} - 2\right)\right]$$

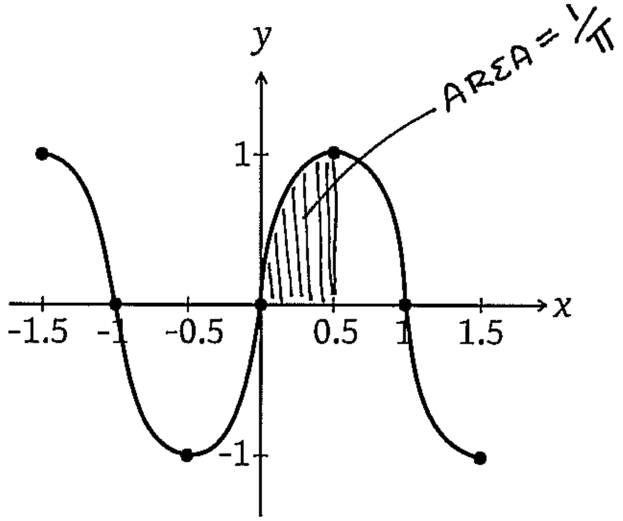
$$\approx \boxed{2.72}$$

Question 4. *(2 marks each = 10 marks)*

a. Evaluate the definite integral $\int_0^{0.5} \sin(\pi x)\, dx$

$$= \frac{1}{\pi}(-\cos \pi x)\Big|_0^{0.5} = \frac{1}{\pi}\left(-\cos(0.5\pi) - (-\cos 0)\right) = \boxed{\frac{1}{\pi}}$$

Using the graph of $y = \sin(\pi x)$ illustrated below. Evaluate the following definite integrals



b. $\int_0^1 \sin(\pi x)\, dx = 2\left(\frac{1}{\pi}\right) = \boxed{2/\pi}$

c. $\int_{-1}^1 \sin(\pi x)\, dx = \boxed{0}$

d. $\int_{-0.5}^1 \sin(\pi x)\, dx = \boxed{\frac{1}{\pi}}$

e. $\int_{-1.5}^1 \sin(\pi x)\, dx = \boxed{\frac{1}{\pi}}$

Question 5. *(5 marks)*
The voltage across an 8.5 nF capacitor in an FM receiver circuit is zero. Find the voltage after 4.00 μs if a current (in mA) $i = 0.042t$ charges the capacitor. (*Make sure to keep track of units*)

$$V_c = \frac{q}{C} = \frac{\int i\,dt}{C}$$

$$V_c = \frac{1}{8.5n} \int 0.042t\,dt$$

$$V_c = \frac{1}{8.5n} (0.021\,t^2 m) + C$$

current voltage (at $t=0$) is 0 so $C = 0$

$$V_c = \frac{1}{8.5n} (0.021t^2)m$$

at $t = 4\mu s$

$$V_c = \frac{1}{8.5n} (0.021)(4\mu)^2 m$$

$$= \boxed{39.5\ nV}$$

Question 6. *(5 marks)*
A current $i = \frac{t}{\sqrt{t^2+1}}$ (in A) is sent through an electric dryer circuit containing a previously uncharged 2.0 μF capacitor. How long does it take for the capacitor voltage to reach 120V?

$$V_c = \frac{\int i \, dt}{C}$$

$$= \frac{1}{2\mu} \int \frac{t}{\sqrt{t^2+1}} dt$$

let $U = t^2+1$
$du = 2t \, dt$
$\frac{1}{2} du = t\,dt$

$$= \frac{1}{2\mu} \int \frac{1}{2} U^{-1/2} du$$

$$= \frac{1}{2\mu} \left(\frac{1}{2} \frac{U^{1/2}}{1/2} \right) + C$$

Previously uncharged; $t=0$ $V_c = 0$

$$0 = \frac{1}{2\mu} \sqrt{0^2+1} + C \longrightarrow C = -500\,000$$

$$120 = \sqrt{t^2+1} - 500000$$

$$\boxed{t \approx 21.9\text{ms}}$$