

Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §8.4 #33 Determine whether the series is absolutely convergent, or conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(\arctan n)^n}$$

Let's apply the root test. Let $a_n = \frac{(-1)^n}{(\arctan n)^n}$

$$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}$$

$$= \lim_{n \rightarrow \infty} \sqrt[n]{\frac{(-1)^n}{(\arctan n)^n}}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{\arctan n}$$

$$= \frac{1}{\frac{\pi}{2}}$$

$$= \frac{2}{\pi} < 1$$

∴ Converges absolutely by root test

Question 2. (5 marks) §8.5 #8

Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$$

Let's first determine the radius of convergence.

$$\text{Let } a_n = \frac{x^n}{n3^n}.$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n \rightarrow \infty} \left| \frac{\frac{x^{n+1}}{(n+1)3^{n+1}}}{\frac{x^n}{n3^n}} \right| = \lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n3^n}{x^n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{x}{3} \cdot \frac{n}{n+1} \right| \\ &= \left| \frac{x}{3} \right| \lim_{n \rightarrow \infty} \frac{n}{n+1} \\ &= \left| \frac{x}{3} \right| \end{aligned}$$

$$\begin{aligned} \therefore \left| \frac{x}{3} \right| &< 1 \\ |x| &< 3 = R \end{aligned}$$

\therefore Radius of convergence is 3.

Let's verify the endpoint of the interval $(-3, 3)$ for convergence.

Let $x = -3$

$$\begin{aligned} &\sum_{n=1}^{\infty} \frac{(-3)^n}{n3^n} \\ &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \end{aligned}$$

Let $b_n = \frac{1}{n}$

$\bullet \lim_{n \rightarrow \infty} b_n = 0$

$\bullet b_{n+1} \stackrel{?}{\leq} b_n$
 $\frac{1}{n+1} \stackrel{?}{\geq} \frac{1}{n}$
 $n \leq n+1$

\therefore Converges at $x = -3$
by alternating series test

Let $x = 3$

$$\begin{aligned} &\sum_{n=1}^{\infty} \frac{(3)^n}{n3^n} \\ &= \sum_{n=1}^{\infty} \frac{1}{n} \end{aligned}$$

\therefore diverges since harmonic series

\therefore interval of convergence is $[-3, 3)$