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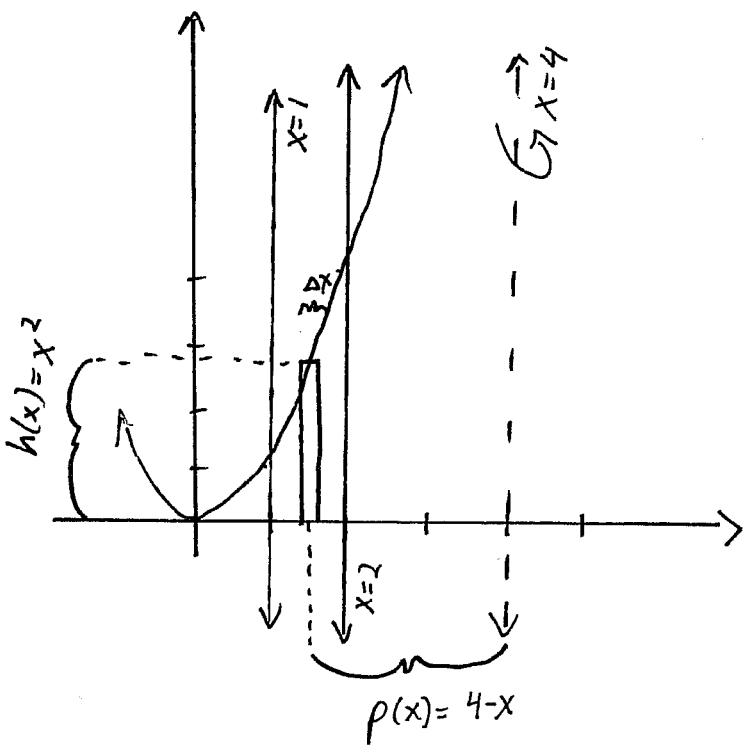
Quiz 9

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §7.3 #17
Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis. Sketch the region and a representative rectangle.

$y = x^2$, $y = 0$, $x = 1$, $x = 2$; about $x = 4$



representative element:

$$\Delta V = 2\pi \rho(x) h(x) \Delta x$$
$$= 2\pi (4-x) x^2 \Delta x$$

$$V = \int_1^2 2\pi (4-x)x^2 dx$$

$$= 2\pi \int_1^2 4x^2 - x^3 dx$$

$$= 2\pi \left[\frac{4x^3}{3} - \frac{x^4}{4} \right]_1^2$$

$$= 2\pi \left[\left[\frac{4 \cdot 2^3}{3} - \frac{2^4}{4} \right] - \left[\frac{4}{3} - \frac{1}{4} \right] \right]$$

$$= 2\pi \left[\left[\frac{32}{3} - 4 \right] - \left[\frac{4}{3} - \frac{1}{4} \right] \right]$$

$$= \frac{67}{6}\pi$$

Question 2. *(2 marks)* §8.1 #6 Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \cdots\right\}$$

$$a_n = (-1)^n \frac{n}{(n+1)^2}$$

Question 3. *(3 marks)* §8.1 #19 Determine whether the sequence converges or diverges. If it converges, find the limit.

$\{n^2 e^{-n}\}$ Let $f(x) = x^2 e^{-x}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^2 e^{-x} \quad \text{I.F. } \infty \cdot 0$$

$$= \lim_{x\to\infty} \frac{x^2}{e^x} \quad \text{I.F. } \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{2x}{e^x} \quad \text{by } H^{\uparrow} \text{ I.F. } \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{2}{e^x} \quad \text{by } H^{\uparrow}$$

$$= 0$$

$\therefore$ i.e. $\{n^2 e^{-n}\} \to 0$ as $n \to \infty$

Bonus. *(5 marks)* Evaluate the improper integral or show it diverges:

$$\int_{-\infty}^{\infty} \frac{1}{2x^2-4x+4}\,dx$$

see test #2