Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S4: Winter 2011

Name: ____________________
Student ID: ____________________

Test 1

This test is graded out of 43 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulae:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(5 marks)* Evaluate using the definition of the definite integral

$$\int_2^3 -6x^2 + 4x - 2\,dx.$$

Question 2. *(5 marks)* Evaluate the definite integral:

$$\int_{\pi/4}^{\pi/3} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta$$

Question 3. *(5 marks)* Evaluate the definite integral:

$$\int_1^2 (z^2+1)\sqrt[3]{z-1}\,dz$$

Question 4. *(5 marks)* Estimate the average value of the function

$$f(x) = \operatorname{arcsec}(x)$$

on the interval $[1,5]$ using two rectangles and the Midpoint Rule. Sketch the curve and approximating rectangles.

Question 5. *(5 marks)* Evaluate the expression and simplify:

$$\frac{d}{dx}\left[\int_{\pi+x}^{\cot 3x} (\pi - u)(\operatorname{arccot} u)^7 \, du\right]$$

Question 6. *(5 marks)* Evaluate the indefinite integral:

$$\int x \ln\left(x^2+1\right)\, dx$$

Question 7. *(5 marks)* Suppose that $f(1)=1, f(2)=2, f'(1)=3, f'(2)=4$ and f'' is continuous. Find the value of $\int_1^2 x f''(x)\, dx$.

Question 8. *(5 marks)* Prove: If $f(x)$ is an even integrable function on $[-a,a]$ then

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

Question 9. *(3 marks)* Prove: If $f(x)$ is a continous function and c is a constant then

$$\int_{a}^{b} cf(x)\,dx = c\int_{a}^{b} f(x)\,dx$$

Bonus Question.
Prove: If f is a continuous on [a,b], then the function g defined by

$$g(x) = \int_a^x f(t)\,dt \quad a \le x \le b$$

is an antiderivative of f, that is, g'(x)=f(x) for $a < x < b$

a. *(1 mark)* Use the limit definition of the derivative,

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

and simplify the numerator of quotient using integral properties.

b. *(1 mark)* Suppose $h > 0$. Since f is continous on $[x, x+h]$ there exists values x_m and x_M such that $f(x_m)$ and $f(x_M)$ are the minimum and maximum of the function, respectively on $[x, x+h]$. Use $f(x_m)$ and $f(x_M)$ to bound above and below the quotient of the limit of part a.

c. *(1 mark)* Assume the inequality is also true for $h < 0$. Apply the Squeeze Theorem to complete the proof.