

Name: _____
Student ID: _____

Test 2

This test is graded out of 43 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (3 marks) Evaluate the definite integral:

$$\int_{\pi/12}^{\pi/9} \sin^2 3\alpha \, d\alpha$$

Question 2. (5 marks) Evaluate the improper integral or show it diverges:

$$\int_{-\infty}^{\infty} \frac{1}{2x^2 - 4x + 4} \, dx$$

Question 3. (5 marks) Evaluate the indefinite integral:

$$\int \csc^3 5\theta \cot^3 5\theta d\theta$$

Question 4. (5 marks) Find the length of the curve.

$$y = \ln(\sec x), 0 \leq x \leq \frac{\pi}{4}$$

Question 5. (5 marks) Sketch and find the total area of the region(s) bounded by the graphs of $y = \arctan x$, $y = \frac{\pi}{4}x$.

Question 6. (5 marks) Evaluate the indefinite integral:

$$\int \frac{3x^2 + 3x + 2}{x^3 + 2x} dx$$

Question 7. (5 marks) Set up the integral to find the volume of the solid obtained from the region bounded by the graphs of $y = (x+1)^2 + 1$, $y = \sqrt{x+1} + 1$ rotated about the line $y = 2$.

Question 8. (5 marks) Evaluate the indefinite integral:

$$\int \frac{81\sqrt{9x^2 + 81}}{x^6} dx$$

Question 9. (5 marks) For what values of m do the line $y = mx$ and the curve

$$y = \frac{x}{x^2 + 1}$$

enclose a region? Find the area of the region.

Bonus Question. (5 marks) Evaluate the limit.

$$\lim_{h \rightarrow 0} \frac{\int_1^{\cos h} \arctan x \, dx}{3e^{-h} - \ln(h+1) + \sin(4h) - 3}$$