

Name: _____
Student ID: _____

Test 3

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Set up the integral to find the volume of the solid obtained from the region bounded by the graphs of $y = -(x-4)^2 + 16$, $y = (x-3)^2 - 9$ rotated about the line $x = -\pi$.

Question 2. (5 marks) Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \left[\sec\left(\frac{1}{n}\right) \right]^n$$

Question 3. (5 marks) Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{1 + 3^{n-1}}{\pi^{n+1}}$$

Question 4. (5 marks) Determine whether the series is absolutely convergent, or conditionally convergent, or divergent.

$$\sum_{n=5}^{\infty} \frac{n5^n(-1)^n}{n!}$$

Question 5. (5 marks) Determine whether the series is absolutely convergent, or conditionally convergent, or divergent.

$$\sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{\sqrt[n]{n}}$$

Question 6. (5 marks) Determine whether the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{\sqrt{n^3 + n^4 + 10}}{\sqrt[3]{n^9 + 3n^4 + 2}}$$

Question 7. (5 marks) Determine whether the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{\operatorname{arcsec}(n)}{n\sqrt{n^2-1}}$$

Question 8. (5 marks) Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n2^n}$$

Question 9. (5 marks) Find the Taylor series for $f(x) = \frac{1}{x^2}$ centered at $x = 2$. Assume that f has a power series expansion. Do not show that $R_n \rightarrow 0$.

Bonus Question. (5 marks) Show that the series diverges.

$$\sum_{n=1}^{\infty} \frac{\int_1^{\cos(\frac{1}{n})} \arctan x \, dx}{3e^{-1/n} - \ln\left(\frac{1}{n} + 1\right) + \sin\left(\frac{4}{n}\right) - 3}$$