Calculus II

Recall that calculus was developed by Isaac Newton and Gottfried Wilhelm Leibniz in order to solve two main problems.

The first problem was investigated in calculus I: Find the tangent line to a curve at a given point on the curve.

In calculus I we found that the slope of the tangent line T to the curve at $x = a$ is given by $f'(a)$, the derivative at a.

The second problem will be investigated in calculus II: Find the area of a planar region bounded by a curve.

Like how the tangent line problem is related to the derivative of a function, we will see that the area problem is related to the antiderivative of a function.

Antiderivatives and the rules of Integration

A function F is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

Example: Show that $F(x) = \frac{1}{3}x^3 + 2x^2 - 5$ is an antiderivative of $f(x) = x^2 + 4x$.
Solution:

Notice that

$$\begin{aligned} F(x) &= \tfrac{1}{3}x^3 + 2x^2 - 5 \\ H(x) &= \tfrac{1}{3}x^3 + 2x^2 + 6 \\ P(x) &= \tfrac{1}{3}x^3 + 2x^2 \end{aligned}$$

are all antiderivatives of $f(x)$.

In fact, if $G(x)$ is any antiderivative of a function $f(x)$ then any antiderivative $F(x)$ of $f(x)$ must be of the form $F(x) = G(x) + C$ where C is a constant.

The operation of finding all antiderivatives is called antidifferentiation (or indefinite integration) and is denoted by the integral sign $\int$

If $F(x)$ is an antiderivative of $f(x)$ then

$$\int f(x)dx = F(x) + C$$

Note: The $\int$ and the dx are paired together. The notation requires **both**.

Example:

$$\int 5x^4 dx$$

Basic Rules of Integration

1. $\int kdx$

Ex: $\int 3dx$

2. $\int x^n dx$

Ex: $\int x^3 dx$

3. $\int cf(x)dx$

Ex: $\displaystyle\int 9x^3 dx$

4. $\int [f(x) \pm g(x)]dx$

Ex: $\displaystyle\int \left(3x^5 - 4x^{3/2} + 2x^{-3}\right) dx$

5. $\int e^x dx$

6. $\int \ln|x| dx$

Ex: **(a)** $\displaystyle\int \left(3x^4 - 5e^x + \frac{1}{x^2} - \frac{1}{x}\right) dx$

(b) $\int \left(6x^3 - \frac{3}{\sqrt{x}} + \sqrt{x}\right) dx$

(c) $\int \frac{x^4 - x + 1}{x^2} dx$

(d) $\int (x^2 + 2x)(x - 1) dx$