

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 9

Question 1. (5 marks) Evaluate the following integral:

$$I = \int_0^{\infty} x e^{-2x} dx = \lim_{b \rightarrow \infty} \int_0^b x e^{-2x} dx$$

$$\begin{aligned} \int x e^{-2x} dx &= -\frac{1}{2} x e^{-2x} + \int \frac{e^{-2x}}{2} dx \\ &= -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C \end{aligned}$$

$\begin{aligned} \text{let } u &= x & dv &= e^{-2x} dx \\ da &= dx & v &= -\frac{e^{-2x}}{2} \end{aligned}$

$$\therefore I = \lim_{b \rightarrow \infty} \left[-\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} \right]_0^b$$

$$= \lim_{b \rightarrow \infty} \left[-\frac{1}{2} b e^{-2b} - \frac{1}{4} e^{-2b} + 0 + \frac{1}{4} e^0 \right]$$

$$\lim_{b \rightarrow \infty} b e^{-2b} = \lim_{b \rightarrow \infty} \frac{b}{e^{2b}} = \text{l.f. } \frac{\infty}{\infty} \stackrel{(H)}{=} \lim_{b \rightarrow \infty} \frac{1}{2e^{2b}} = 0$$

$$\therefore I = 0 - 0 + 0 + \frac{1}{4}$$

$$= \frac{1}{4}$$

Question 2. (5 marks) Solve the first-order differential equation:

$$y' = \frac{y \ln x}{x}$$

$$\frac{dy}{dx} = \frac{y \ln x}{x} \Rightarrow \frac{1}{y} dy = \frac{\ln x}{x} dx$$

$$\therefore \int \frac{1}{y} dy = \int \frac{\ln x}{x} dx = \int u du$$

$$\text{LET } u = \ln x \\ du = \frac{1}{x} dx$$

$$\ln|y| = \frac{u^2}{2} + c$$

$$\ln|y| = \frac{(\ln x)^2}{2} + c$$

$$e^{\ln|y|} = e^{\frac{(\ln x)^2}{2} + c}$$

$$|y| = e^{\frac{(\ln x)^2}{2}} \cdot e^c$$

$$|y| = |c| e^{\frac{(\ln x)^2}{2}}$$

$$y = \pm |c| e^{\frac{(\ln x)^2}{2}}$$

$$y = c_1 e^{\frac{(\ln x)^2}{2}}$$