

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 6

Question 1. (10 marks) In a study conducted by a certain country's Economic Development Board, it was found that the Lorentz curve for the distribution of income of college teachers and lawers was described by the functions

$$f(x) = \frac{13}{14}x^2 + \frac{1}{14}x \quad \text{and} \quad g(x) = \frac{9}{11}x^4 + \frac{2}{11}x$$

respectively.

(a) Compute the coefficient of inequality for each Lorentz curve.

(b) Which profession has a more equitable income distribution?

$$\begin{aligned} L_f &= 2 \int_0^1 [x - f(x)] dx = 2 \int_0^1 \left[x - \left(\frac{13}{14}x^2 + \frac{1}{14}x \right) \right] dx = 2 \int_0^1 \left[-\frac{13}{14}x^2 + \frac{13}{14}x \right] dx \\ &= \frac{13}{7} \int_0^1 -x^2 + x dx = \frac{13}{7} \left[-\frac{x^3}{3} + \frac{x^2}{2} \right]_0^1 = \frac{13}{7} \left[-\frac{1}{3} + \frac{1}{2} - 0 \right] \\ &= \frac{13}{7} \left(\frac{1}{6} \right) = \frac{13}{42} \approx 0.3095 \end{aligned}$$

$$\begin{aligned} L_g &= 2 \int_0^1 [x - g(x)] dx = 2 \int_0^1 \left[x - \left(\frac{9}{11}x^4 + \frac{2}{11}x \right) \right] dx \\ &= 2 \int_0^1 \left(\frac{9}{11}x - \frac{9}{11}x^4 \right) dx = \frac{18}{11} \int_0^1 (x - x^4) dx = \frac{18}{11} \left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1 \\ &= \frac{18}{11} \left[\frac{1}{2} - \frac{1}{5} - 0 \right] = \frac{18}{11} \left(\frac{3}{10} \right) = \frac{54}{110} \approx 0.4909 \end{aligned}$$

b) SINCE $L_f \approx 0.3095 < L_g \approx 0.4909$ TEACHERS
HAVE A MORE EQUITABLE INCOME DISTRIBUTION.