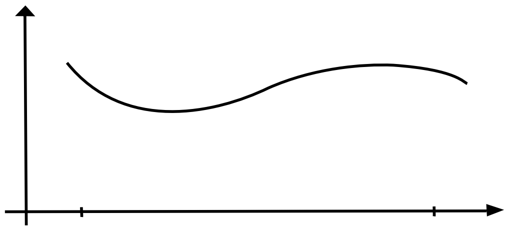
Geometric Interpretation of the Definite Integral

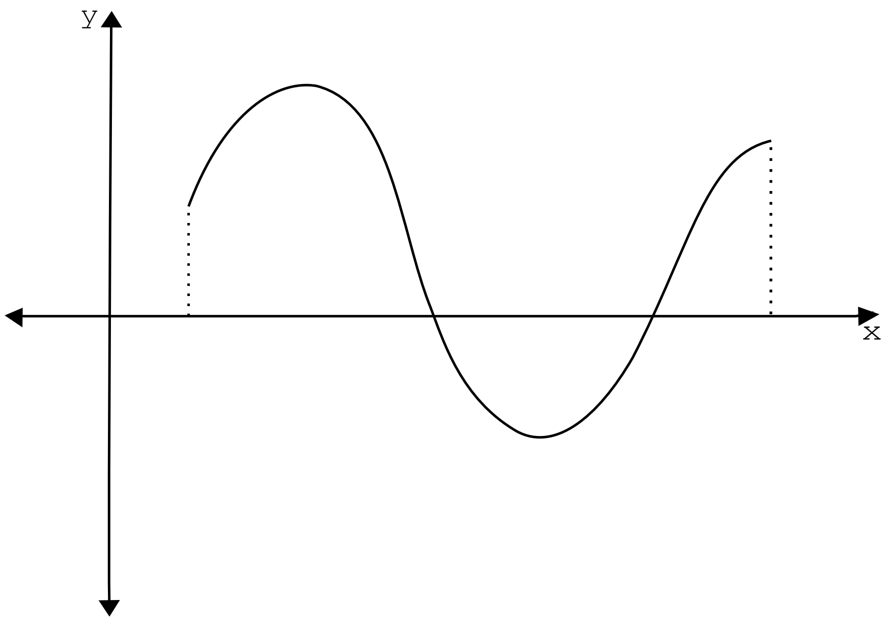
Recall that, if $f(x)$ is a continuous funtion, the definition of the definite integral is the following:

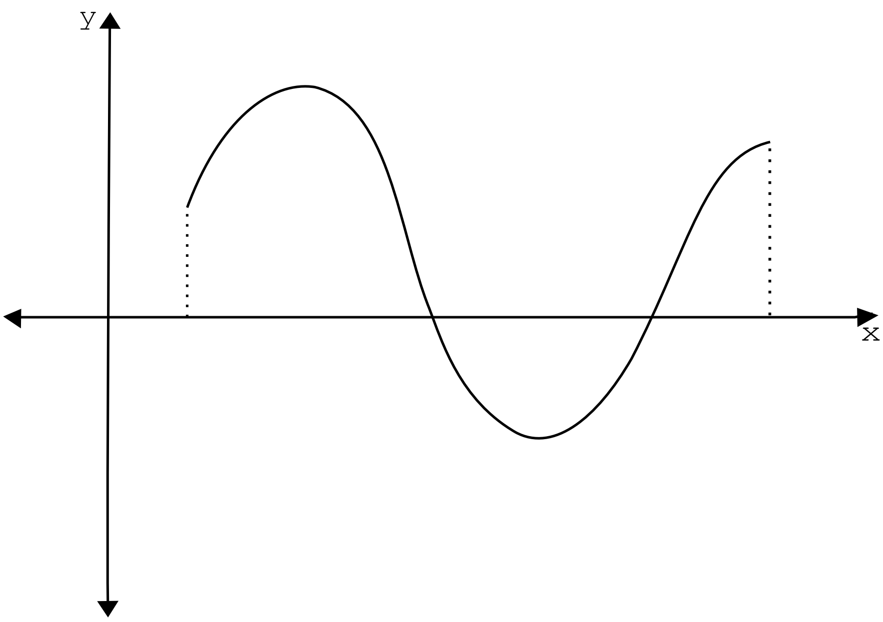
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

If $f(x)$ is also a positive function on the interval $[a,b]$ then this integral gives us the area under $f(x)$ and above the x-axis between a and b.



But what if $f(x)$ is not a positive function on $[a,b]$? What does the definite integral represent?





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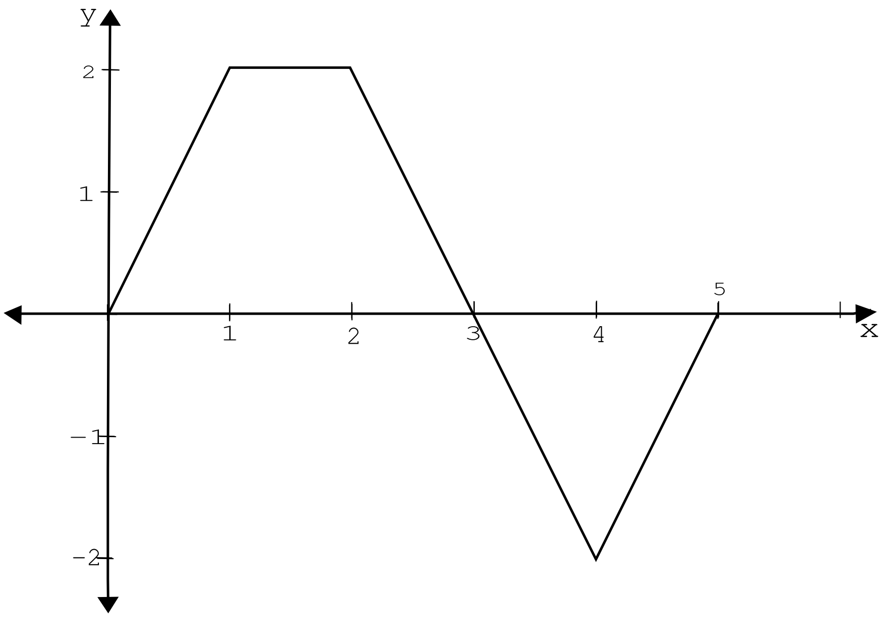
If $f(x)$ is a continuous function on $[a,b]$ then

$$\int_a^b f(x)\,dx$$

is equal to the area above $[a,b]$ minus the area below $[a,b]$.

Example: Let $f(x)$ be the funtion with graph



Find:

(a) $\int_0^1 f(x)\,dx$

(b) $\int_0^0 f(x)\,dx$

(c) $\int_0^3 \int f(x)\,dx$

(d) $\int_0^5 f(x)\,dx$

Example: Let $f(x) = 4 - x^2$

(a) Graph $f(x)$

(b) Evaluate $\int_0^2 f(x)\,dx$

(c) Evaluate $\int_2^3 f(x)\,dx$

(d) Evaluate $\int_0^3 f(x)\,dx$