L'Hôpital's Rule

Note: In what follows a can represent a (finite) real number or ∞ or $-\infty$.

$$\text{If} \quad \lim_{x \to a} f(x) = 0 \quad \text{and} \quad \lim_{x \to a} g(x) = 0 \quad \text{then} \quad \lim_{x \to a} \frac{f(x)}{g(x)}$$

may or may not exist, and we call this an **indeterminate form of type** $\frac{0}{0}$.

$$\text{Similarly, if} \quad \lim_{x \to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x \to a} g(x) = \pm\infty \quad \text{then} \quad \lim_{x \to a} \frac{f(x)}{g(x)}$$

may or may not exist, and we call this limit an **indeterminate form of type** $\frac{\infty}{\infty}$.
For limits of these two types we can apply the following rule.

L'Hôpital's Rule
Suppose f and g are differentiable functions and that

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

is and indeterminate form of the type $\frac{0}{0}$ or $\frac{\infty}{\infty}$. Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

provided that the limit on the right exists (or is ∞ or $-\infty$).

Examples: Find the following limits if they exist.

1) $\displaystyle \lim_{x \to 2} \frac{5x^3 - 13x^2 + 6x}{4x^3 - 13x + 10}$

2) $\lim\limits_{x \to \infty} \dfrac{15x - 1}{3x + 2}$

3) $\lim\limits_{x \to \infty} \dfrac{10x + 5}{3x^2 - 7x + 4}$

4) $\lim\limits_{x \to \infty} \dfrac{9x^2 + 3x - 7}{2x^2 - 5x + 1}$

5) $\lim_{x \to 5} \frac{x+2}{3x-7}$

6) $\lim_{x \to 0} \frac{e^x}{1-\cos x}$

7) $\lim_{x \to 0} \frac{\cos x - 1}{e^x - 1}$

8) $\lim_{x \to 1^+} \frac{7\sqrt{x-1}}{\sin(x-1)}$

9) $\lim_{x \to \infty} \frac{3\ln(5x+3)}{2\ln(x+4)}$