SOLUTIONS #
NYA ELECTRO
WINTER 2012

[Q1] Identify the values of a

(a) $\int \frac{7}{3x^6}\,dx = a x^{-5} + C$

$$\int \frac{7}{3x^6}\,dx = \int \frac{7}{3} x^{-6}\,dx = \frac{7}{3}\frac{x^{-5}}{-5} + C = \frac{7}{-15} x^{-5} + C$$

$$\boxed{a = -7/15}$$

(b) $\int -6x^3\,dx = a x^4 + C$

$$\int -6x^3\,dx = -\frac{6x^4}{4} + C \qquad \boxed{a = -3/2}$$

(c) $\int 5x^{-3/2}\,dx = \frac{a}{\sqrt{x}} + C$

$$\int 5x^{-3/2}\,dx = \frac{5x^{-1/2}}{-1/2} + C$$

$$= \frac{-10}{\sqrt{x}} + C \qquad \boxed{a = -10}$$

[Q2]

(a) $\int \frac{-2\cos(4x)}{\sin(4x)}\,dx$

$u = \sin(4x)$

$du = \cos(4x) \cdot 4\,dx$

$\frac{1}{4}\,du = \cos(4x)\,dx$

$= \int \frac{-2\ (\frac{1}{4})}{u}\,du$

$= \int -\frac{1}{2}\frac{1}{u}\,du = -\frac{1}{2}\ln|u| + C = \boxed{-\frac{1}{2}\ln|\sin(4x)| + C}$

(b) $\int (4x^3-6x)(4x^4-12x^2)^{-5}\,dx$

$u = 4x^4 - 12x^2$

$du = 16x^3 - 24x\,dx$

$du = 4(4x^3-6x)\,dx$

$\frac{1}{4}\,du = (4x^3-6x)\,dx$

$= \int u^{-5}\,\frac{1}{4}\,du$

$= \frac{1}{4}\,\frac{u^{-4}}{-4} + C$

$= \frac{-1}{16}u^{-4} + C = \boxed{-\frac{1}{16}(4x^4-12x^2)^{-4} + C}$

(c) $\int \frac{\sec^2 t}{\sqrt{5\tan t}}\,dt$

$u = 5\tan t$

$du = 5\sec^2 t\,dt$

$\frac{1}{5}\,du = \sec^2 t\,dt$

$= \int \frac{1}{5}\,\frac{1}{\sqrt{u}}\,du$

$= \frac{1}{5}\,\frac{u^{1/2}}{1/2} + C = \boxed{\frac{2}{5}\sqrt{5\tan t} + C}$

(d) $\int_0^3 x\sqrt{x+1}\,dx$

$u = x+1$
$du = dx$

$u-1 = x$

$x=0 \quad u=1$
$x=3 \quad u=4$

$$= \int_1^4 (u-1)\sqrt{u}\,du$$

$$= \int_1^4 u^{3/2} - u^{1/2}\,du$$

$$= \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\Big|_1^4$$

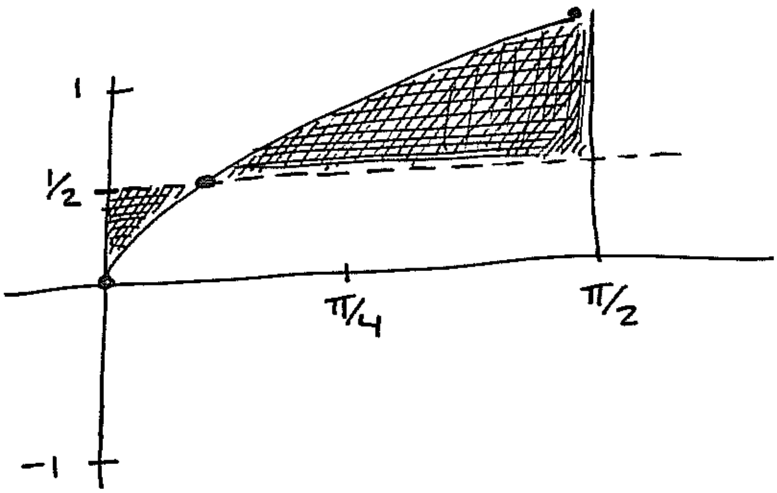
$$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\Big|_1^4 = \left[\tfrac{2}{5}(4)^{5/2} - \tfrac{2}{3}(4)^{3/2}\right] - \left[\tfrac{2}{5}(1)^{5/2} - \tfrac{2}{3}(1)^{3/2}\right]$$

$$= \left[\tfrac{2}{5}(2)^5 - \tfrac{2}{3}(2)^3\right] - \left[\tfrac{2}{5} - \tfrac{2}{3}\right]$$

$$= \left[\frac{64}{5} - \frac{16}{3}\right] - \left[\tfrac{2}{5} - \tfrac{2}{3}\right]$$

$$= \left(\frac{116}{15}\right)$$

[Q3] FIND THE AREA BETWEEN THE CURVES
$y = \sin x$ & $y = 1/2$ ON THE interval
$0 \le x \le \pi/2$



INTERSECTION $y = 1/2$ $y = \sin x$

$$\sin x = 1/2$$
$$x = \sin^{-1}(1/2) = \pi/6$$

$$\text{AREA} = \int_0^{\pi/6} 1/2 - \sin x \, dx + \int_{\pi/6}^{\pi/2} \sin x - 1/2 \, dx$$

$$= \frac{1}{2} + \cos x \Big|_0^{\pi/6} + -\cos x - 1/2\, x$$

$$= \left[\frac{1}{2}(\pi/6) + \cos(\pi/6)\right] - \left[1/2(0) + \cos(0)\right]$$

$$= \pi/12 + \cos(\pi/6) - 1$$