Assignment #2 Solutions
Calculus for Electronics Engineering

[Question 1] Evaluate the following limits

(1) $$\lim_{x\to 4} \frac{\sqrt{x-4}}{x^2-16}$$

$$= \lim_{x\to 4} \frac{\sqrt{x-4}\,\sqrt{x-4}}{(x-4)(x+4)\sqrt{x-4}}$$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}}{\cancel{(x-4)}(x+4)(\sqrt{x-4})}$$

$$= \lim_{x\to 4} \frac{1}{(x+4)\sqrt{x-4}}$$

Tends to "1/0"
Vertical asymptote
Limit DNE

$$\lim_{x\to 4^-} \frac{1}{(x+4)(\sqrt{x-4})} \quad \text{DNE}$$

(negative under square root)

$$\lim_{x\to 4^+} \frac{1}{(x+4)\sqrt{x-4}} \longrightarrow +\infty$$

(2) $$\lim_{x\to\infty} \frac{2x^5-3}{-3x^4-2x^2}$$

$$= \lim_{x\to\infty} \frac{2x^5/x^4 - 3/x^4}{-3x^4/x^4 - 2x^2/x^4}$$

$$= \lim_{x\to\infty} \frac{2x - 3/x^4}{-3 - 2/x^2}$$

$$= \lim_{x\to\infty} 2x/-3$$

$$\longrightarrow -\infty$$

(3) $$\lim_{x\to -\infty} \frac{1+2x^2-3x^7}{-3x^4+7x^7}$$

$$= \lim_{x\to -\infty} \frac{1/x^7 + 2x^2/x^7 - 3x^7/x^7}{-3x^4/x^7 + 7x^7/x^7}$$

$$= \lim_{x\to -\infty} \frac{1/x^7 + 2/x^5 - 3}{-3/x^3 + 7}$$

$$= -3/7$$

[QUESTION 2]

$$f(x) = \begin{cases} x-2 & x>1 \\ -1 & x=1 \\ -x^2+2x-2 & x<1 \end{cases}$$

(i) $\lim_{x\to 1^+} f(x)$

$= \lim_{x\to 1^+} x-2$

$= -1$

(ii) $\lim_{x\to 1^-} f(x)$

$= \lim_{x\to 1^-} -x^2+2x-2$

$= -(1)^2+2(1)-2$

$= -1$

(iii) Since $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = -1$

THEN $\lim_{x\to 1} f(x) = -1$

(iv) $f(1) = -1$

[QUESTION 3]

For each of the following functions, find the slope of the tangent line at $x=-1$
$x=0$
$x=2$
$x=7$

(i) $f(x) = 2x-3x^2$

$$m = \lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{[2(x+\Delta x)-3(x+\Delta x)^2]-[2x-3x^2]}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{\cancel{2x}+2\Delta x-\cancel{3x^2}-6x\Delta x-3(\Delta x)^2-\cancel{2x}+\cancel{3x^2}}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{2\Delta x-6x\Delta x-3(\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\,(2 - 6x - 3\Delta x)}{\cancel{\Delta x}} = 2 - 6x$$

So THE slopes are:

AT $x = -1$ $\quad m = 8$

$x = 0$ $\quad m = 2$

$x = 2$ $\quad m = -10$

$x = 7$ $\quad m = -40$

(ii) $f(x) = \sqrt{x-3}$

$$m = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x-3} - \sqrt{x-3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left[\frac{\sqrt{x+\Delta x-3} - \sqrt{x-3}}{\Delta x}\right]\left[\frac{\sqrt{x+\Delta x-3} + \sqrt{x-3}}{\sqrt{x+\Delta x-3} + \sqrt{x-3}}\right]$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x}+\Delta x-\cancel{3}-\cancel{x}+\cancel{3}}{\Delta x\left(\sqrt{x+\Delta x-3}+\sqrt{x-3}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x-3}+\sqrt{x-3}}$$

$$= \frac{1}{2\sqrt{x-3}}$$

AT $x = -1$ $\quad m = \text{DNE}$

$x = 0$ $\quad m = \text{DNE}$

$x = 2$ $\quad m = \text{DNE}$

$x = 7$ $\quad m = \frac{1}{2\sqrt{4}} = 1/4$

(iii) $f(x) = \dfrac{1}{x+2}$

$$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x+\Delta x+2} - \dfrac{1}{x+2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+2) - (x+\Delta x+2)}{(x+\Delta x+2)(x+2)} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\cancel{\Delta x}}{(x+\Delta x+2)(x+2)(\cancel{\Delta x})}$$

$$= -\frac{1}{(x+2)^2}$$

AT $x = -1$ $m = -1$

$x = 0$ $m = -1/4$

$x = 2$ $m = -1/16$

$x = 7$ $m = -1/81$