In-Class Assignment #3
SOLUTIONS
Feb 6th 2012
NYA Calculus (Electronics)

① $f(x) = x^2 + 5x$

a. $f(3) = 9 + 15 = 24$

b. $f(-2) = 4 - 10 = -6$

c. $f(y) = y^2 + 5y$

d. $f(x+\Delta x) = (x+\Delta x)^2 + 5(x+\Delta x)$
$= x^2 + 2x\Delta x + (\Delta x)^2 + 5x + 5\Delta x$

e. $f(x^3) = (x^3)^2 + 5x^3$
$= x^6 + 5x^3$

f. $\dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

$= \dfrac{\left[x^2 + 2x\Delta x + (\Delta x)^2 + 5x + 5\Delta x\right] - \left[x^2 + 5x\right]}{\Delta x}$

$= \dfrac{2x\Delta x + (\Delta x)^2 + 5\Delta x}{\Delta x}$

$= \dfrac{\Delta x\,(2x + \Delta x + 5)}{\Delta x}$

② $f(x) = \sqrt{5 - x} + 2$

a. $f(1) = \sqrt{5-1} + 2 = 4$

b. $f(-4) = \sqrt{5-(-4)} + 2 = 5$

c. $f(y) = \sqrt{5-y} + 2$

d. $f(x+\Delta x) = \sqrt{5-(x+\Delta x)} + 2$

e. $f(x^3) = \sqrt{5 - x^3} + 2$

f. $\dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \dfrac{\left[\sqrt{5-(x+\Delta x)} + 2\right] - \left[\sqrt{5-x} + 2\right]}{\Delta x}$

$= \dfrac{\sqrt{5-(x+\Delta x)} - \sqrt{5-x}}{\Delta x}$

(3) $f(x) = \frac{2}{1+x}$

a. $f(1) = \frac{2}{2} = 1$

b. $f(-4) = \frac{2}{1-4} = -2/3$

c. $f(y) = \frac{2}{1+y}$

d. $f(x+\Delta x) = \frac{2}{1+x+\Delta x}$

e. $f(x^3) = \frac{2}{1+x^3}$

f. $\frac{f(x+\Delta x) - f(x)}{\Delta x}$

$= \frac{\frac{2}{1+x+\Delta x} - \frac{2}{1+x}}{\Delta x}$

(4) $f(x) = e^x/x$

a. $f(1) = \frac{e^1}{1} = e$

b. $f(-4) = e^{-4}/-4 = \frac{1}{-4e^4}$

c. $f(y) = e^y/y$

d. $f(x+\Delta x) = \frac{e^{x+\Delta x}}{x+\Delta x}$

e. $f(x^3) = \frac{e^{x^3}}{x^3}$

f. $\frac{f(x+\Delta x) - f(x)}{\Delta x}$

$= \frac{\frac{e^{x+\Delta x}}{x+\Delta x} - \frac{e^x}{x}}{\Delta x}$

(5) [QUESTION 1]

$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\Delta x(2x+\Delta x+5)}{\Delta x}$

$= 2x + 5$

slope @

a. $x=1$ $m=7$

b. $x=2$ $m=9$

c. $x=-2$ $m=1$

d. $x=5$ $m=15$

[QUESTION 2]

$$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left[\frac{\sqrt{5-(x+\Delta x)} - \sqrt{5-x}}{\Delta x}\right]\left[\frac{\sqrt{5-(x+\Delta x)} + \sqrt{5-x}}{\sqrt{5-(x+\Delta x)} + \sqrt{5-x}}\right]$$

$$= \lim_{\Delta x \to 0} \frac{[5-(x+\Delta x)] - [5-x]}{\Delta x\,[\sqrt{5-(x+\Delta x)} + \sqrt{5-x}]}$$

$$= \lim_{\Delta x \to 0} \frac{-\cancel{\Delta x}}{\cancel{\Delta x}\,[\sqrt{5-(x+\Delta x)} + \sqrt{5-x}]}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{\sqrt{5-(x+\Delta x)} + \sqrt{5-x}}$$

$$= \frac{-1}{\sqrt{5-x} + \sqrt{5-x}} = \frac{-1}{2\sqrt{5-x}}$$

slope @

a. $x = 1$ $\quad m = \frac{-1}{2\sqrt{4}} = -1/4$

b. $x = 2$ $\quad m = \frac{-1}{2\sqrt{3}}$

c. $x = -2$ $\quad m = \frac{-1}{2\sqrt{7}}$

d. $x = 5$ $\quad m = \frac{-1}{2\sqrt{0}}$ DNE

[QUESTION 3]

$$m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{2}{1+x+\Delta x} - \frac{2}{1+x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{2(1+x) - 2(1+x+\Delta x)}{(1+x+\Delta x)(1+x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2+2x-2-2x-2\Delta x}{(1+x+\Delta x)(1+x)} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-2\cancel{\Delta x}}{(1+x+\Delta x)(1+x)} \cdot \frac{1}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} \frac{-2}{(1+x+\Delta x)(1+x)} = \frac{-2}{(1+x)^2}$$

slope @

a. $x = 1$ $\quad \frac{-2}{2^2} = -1/2$

b. $x = 2$ $\quad m = \frac{-2}{3^2} = -2/9$

c. $x = -2$ $\quad m = \frac{-2}{(-1)^2} = -2$

d. $x = 5$ $\quad m = \frac{-2}{6^2} = \frac{-2}{36} = -1/18$