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ASSIGNMENT # 7
(MULTIPLE CHOICE)
DERIVATIVES
SOLUTIONS
MARCH 2012

FUNCTION 1

$$f(x) = \frac{\ln(x^3)}{e^{\sin x \cos x}}$$

Solⁿ.

$$f'(x) = \frac{\frac{1}{x^3} \cdot 3x^2 e^{\sin x \cos x} - e^{\sin x \cos x} (\cos^2 x - \sin^2 x) \ln(x^3)}{(e^{\sin x \cos x})^2}$$

ANSWER (a.)

FUNCTION 2

$$y = (\ln \cos x)^{\cos x} \quad \text{sol}^n.$$

$$\ln y = \ln (\ln \cos x)^{\cos x}$$

$$\ln y = \cos x \ln (\ln \cos x)$$

$$\frac{1}{y} y' = -\sin x \ln (\ln \cos x) + \frac{1}{(\ln \cos x)} \cdot \frac{1}{\cos x} (-\sin x) \cdot \cancel{\cos x}$$

$$y' = (\ln \cos x)^{\cos x} \left[-\sin x \ln (\ln \cos x) + \frac{1}{(\ln \cos x)} (-\sin x) \right]$$

ANSWER (NONE)

FUNCTION 3

$$f(x) = \frac{\ln x \cos x}{\ln(x^2)}$$

Soln.

$$f'(x) = \frac{\left(\frac{1}{x} \cos x - \sin x \ln x \right) (\ln(x^2)) - \frac{2}{x} \ln x \cos x}{(\ln(x^2))^2}$$

ANSWER (b.)

FUNCTION 4

2

$$f(x) = \cos x e^x$$

Soln.

$$\begin{aligned} f'(x) &= -\sin x e^x + e^x \cos x \\ &= e^x (-\sin x + \cos x) \end{aligned}$$

ANSWER (c)

FUNCTION 5

$$f(x) = \ln \left(\frac{x^4 + 5x^2}{3x^2 + 4x} \right)^2$$

Soln.

$$f(x) = 2 \ln(x^4 + 5x^2) - 2 \ln(3x^2 + 4x)$$

$$f'(x) = \frac{2(4x^3 + 10x)}{x^4 + 5x^2} - \frac{2(6x + 4)}{3x^2 + 4x}$$

ANSWER (d)

FUNCTION 6

$$f(x) = \ln(\sin(e^{x^2}))$$

Soln.

$$f'(x) = \frac{1}{\sin(e^{x^2})} \cdot \cos(e^{x^2}) \cdot e^{x^2} \cdot 2x$$

ANSWER (d)

FUNCTION 7

$$f(x) = \ln \left(\frac{e^{\cos x}}{\ln x} \right)$$

Soln.

$$f(x) = \cos x - \ln(\ln x)$$

$$f'(x) = -\sin x - \frac{1}{\ln x} \cdot \frac{1}{x}$$

ANSWER (none)

FUNCTION 8

$$f(x) = \frac{\ln(x^2+x)}{4x^2+2}$$

Soln.

$$f'(x) = \frac{\frac{1}{x^2+x} (2x+1)(4x^2+2) - 8x \ln(x^2+x)}{(4x^2+2)^2}$$

ANSWER (b)

FUNCTION 9

$$e^{x \cos y} = x+y$$

Soln.

$$e^{x \cos y} \cdot (\cos y - (\sin y)y'x) = 1+y'$$

$$\cos y e^{x \cos y} - x \sin y e^{x \cos y} y' = 1+y'$$

$$\cos y e^{x \cos y} - 1 = y' + x \sin y e^{x \cos y} y'$$

$$y' = \frac{\cos y e^{x \cos y} - 1}{1 + x \sin y e^{x \cos y}}$$

ANSWER (a)

FUNCTION 10

$$f(x) = \ln((\sin(x^{55}))^{\cos(x^{55})})$$

Soln.

$$f(x) = \cos(x^{55}) \ln(\sin(x^{55}))$$

$$f'(x) = -\sin(x^{55}) \cdot 55x^{54} \ln(\sin(x^{55})) + \frac{1}{\sin(x^{55})} \cdot \cos^2(x^{55}) \cdot 55x^{54}$$

ANSWER (none)

FUNCTION 11

$$y = (\ln(x^x))^x$$

solⁿ.

$$y = (x \ln x)^x$$

$$\ln y = \ln(x \ln x)^x$$

$$\ln y = x \ln(x \ln x)$$

$$\frac{1}{y} y' = \ln(x \ln x) + \frac{1}{x \ln x} (\ln x + \frac{1}{x} \cdot x) x$$

$$\frac{1}{y} y' = \ln(x \ln x) + \frac{x \ln x + x}{x \ln x}$$

$$\frac{1}{y} y' = \ln(x \ln x) + 1 + \frac{1}{\ln x}$$

$$y' = y \ln(x \ln x) + y + \frac{y}{\ln x}$$

ANSWER (b)

FUNCTION 12

$$f(x) = \ln(e^{\sin(\frac{x}{x^3+26})})^{16}$$

soln.

$$f'(x) = 16(\ln(e^{\sin(\frac{x}{x^3+26})}))^{15} \cdot \frac{1}{e^{\sin(\frac{x}{x^3+26})}} \cdot e^{\sin(\frac{x}{x^3+26})} \cdot \cos(\frac{x}{x^3+26}) \cdot (\frac{x^3+26-3x^2 \cdot x}{(x^3+26)^2})$$

ANSWER (c)

FUNCTION 13

$$f(x) = \frac{e^{\sin x} + 1}{\ln(\sin(x^2))}$$

soln.

$$f'(x) = \frac{e^{\sin x} \cos x \ln(\sin(x^2)) - \frac{1}{\sin(x^2)} \cos(x^2) \cdot 2x \cdot (e^{\sin x} + 1)}{(\ln(\sin(x^2)))^2}$$

ANSWER (b)

FUNCTION 14

$$f(x) = \frac{(e^x)^x}{\ln x} \leftarrow \text{NOTE THAT I forgot brackets (\& they are very important!)}$$

$$\text{Sol}^n. \quad f(x) = \frac{e^{x^2}}{\ln x}$$

$$f'(x) = \frac{e^{x^2} \cdot 2x \cdot \ln x - \frac{1}{x} e^{x^2}}{(\ln x)^2}$$

$$= \frac{2x e^{x^2}}{\ln x} - \frac{e^{x^2}}{x (\ln x)^2}$$

ANSWER (d) (or none b/c of missing brackets)

FUNCTION 15

$$f(x) = \ln \left(\frac{x^x}{(\sin x)^x} \right)$$

Solⁿ.

$$f(x) = x \ln x - x \ln \sin x$$

$$f'(x) = \ln x + x \left(\frac{1}{x} \right) - \ln \sin x - x \left(\frac{1}{\sin x} \right) \cos x$$

$$= \ln x + 1 - \ln \sin x - x \cot x$$

ANSWER (none)

FUNCTION 16

$$f(x) = \tan^{-19}(9x+7)$$

Solⁿ.

$$f'(x) = -19 \tan^{-20}(9x+7) \sec^2(9x+7) \cdot 9$$

ANSWER (b)

Function 17

$$f(x) = \cos(\ln\sqrt{x})$$

Solⁿ.

$$f'(x) = -\sin(\ln\sqrt{x}) \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{-\sin(\ln\sqrt{x})}{2x}$$

ANSWER (d.)

Function 18

$$f(x) = \frac{\sin x}{\sqrt{x}}$$

Solⁿ.

$$f'(x) = \frac{(\cos x)\sqrt{x} - \frac{1}{2\sqrt{x}} \sin x}{x}$$

ANSWER (c.)

Function 19

$$f(x) = (\cos x)e^{x+1}$$

Solⁿ.

$$f'(x) = (-\sin x)e^{x+1} + e^{x+1} \cos x$$

ANSWER (d.)

Function 20

$$y = (\sin x)^x$$

Solⁿ.

$$\ln y = x \ln \sin x$$

$$\frac{1}{y} y' = \ln \sin x + \frac{1}{\sin x} (\cos x)x$$

$$y' = (\sin x)^x (\ln \sin x + x \cot x)$$

ANSWER (a.)

Function 21

$$y = \sqrt[13]{\sin y} + x$$

Solⁿ.

$$y' = \frac{1}{13} \sin y^{-12/13} \cos y y' + 1$$

$$y' = \frac{1}{1 - \frac{1}{13} (\sin y)^{-12/13} \cos y}$$

ANSWER (b.)