NAME: Solutions

TEST 1

DAWSON COLLEGE
NYA-Electrotech Section 7 - Calculus 1
Instructor: E. Richer
Date: March 9th 2012
This test is marked out of **40 points**

Question 1. *(5 marks)*
Sketch a graph that satisfies all of the following conditions:

$\lim_{x\to\infty} f(x) \to \infty$

$\lim_{x\to 2^+} f(x) \to -\infty$

$\lim_{x\to 2^-} f(x) \to \infty$

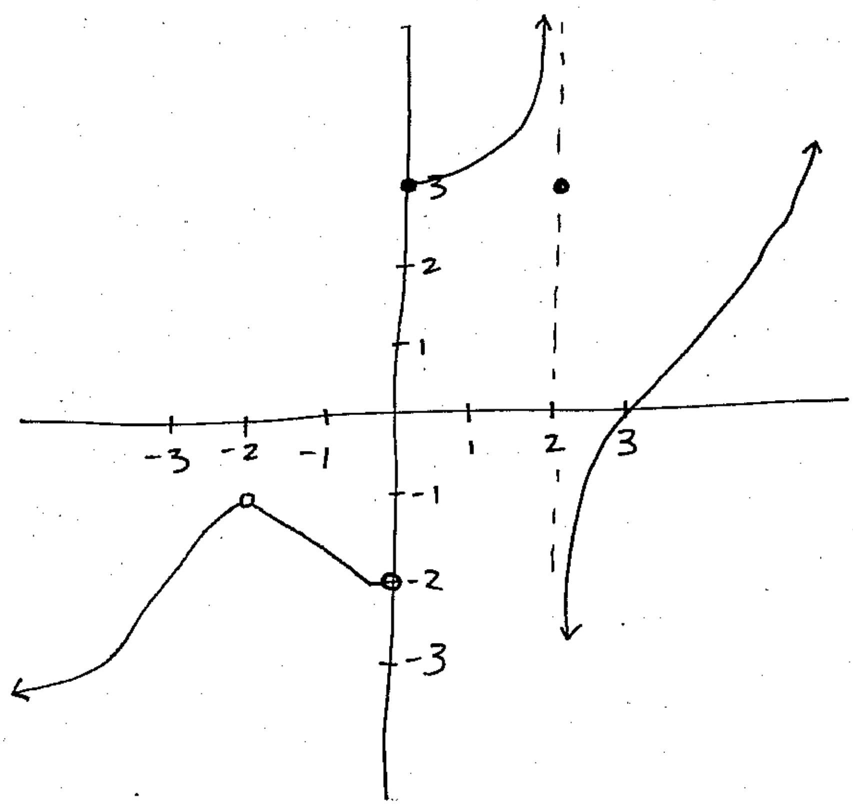
$\lim_{x\to -2} f(x) = -1$

$f(2) = 3$

$\lim_{x\to 0} f(x)$ does not exist

$f(0) = 3$

THERE ARE MANY POSSIBLE SOLUTIONS TO THIS PROBLEM; ONE IS ILLUSTRATED BELOW:



Question 2. *(5 marks)*

(a) Evaluate the following limit:

$$\lim_{x\to 3}\frac{(x^2-5x+6)(x-1)}{(x-3)} = \lim_{x\to 3}\frac{\cancel{(x-3)}(x-2)(x-1)}{\cancel{(x-3)}}$$

$$= (3-2)(3-1)$$

$$= (1)(2) = 2$$

(b) Graph the function $y = f(x) = \frac{(x^2-5x+6)(x-1)}{(x-3)}$ $= \frac{(x-2)(x-1)\,(x-3)}{(x-3)}$

$$= \frac{(x^2-3x+2)(x-3)}{(x-3)}$$

Your graph must include:

- x-intercept(s)
- y-intercept
- the vertex
- any "hollow" dots.

HOLLOW DOT @ $x=3$ $y=2$

VERTEX $x = 3/2$ $y = (3/2)^2 - 3(3/2) + 2$

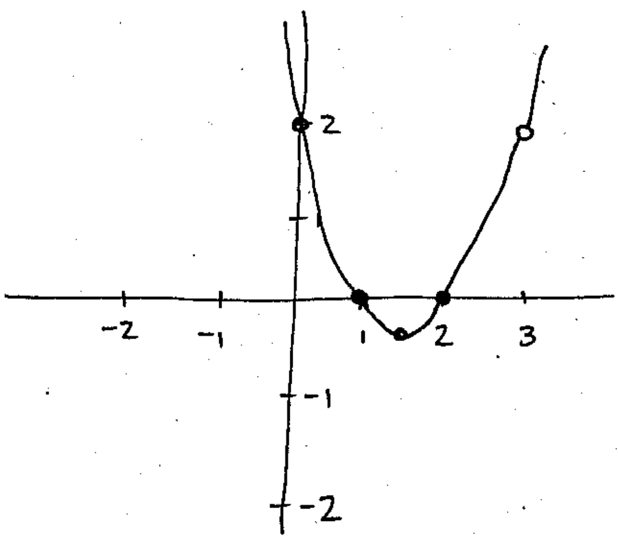
$= 9/4 - 9/2 + 2$

$= 9/4 - 18/4 + 8/4$

$= -1/4$

X-INTERCEPTS (2,0) (1,0)

Y-INTERCEPT (0,2)



Question 3. *(5 marks)*
Evaluate the following limits. If the limit does not exist, determine if its one-sided limits tend to $\pm\infty$.

(a) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$

$$= \lim_{x\to 4} \frac{(x-4)(\sqrt{x}+2)}{(\sqrt{x}-2)(\sqrt{x}+2)} = \lim_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{(x-4)}} = \sqrt{4}+2 = 4$$

(b) $\lim_{x\to 1} \frac{x+2}{x^2+x-2}$ $= \lim_{x\to 1} \frac{\cancel{x+2}}{(\cancel{x+2})(x-1)} = \lim_{x\to 1} \frac{1}{x-1}$ limit DNE

$$\lim_{x\to 1^-} \frac{1}{x-1} \longrightarrow -\infty \qquad \lim_{x\to 1^+} \frac{1}{x-1} \longrightarrow +\infty$$

(c) $\lim_{x\to 0} 3x^2 + x - \sqrt{x+4}$ $= -\sqrt{0+4} = -2$

(d) $\lim_{x\to -1} \frac{x^2+3x+2}{x^2-1}$ $= \lim_{x\to -1} \frac{(x+2)\cancel{(x+1)}}{\cancel{(x+1)}(x-1)} = \lim_{x\to -1} \frac{x+2}{x-1} = \frac{1}{-2}$

(e) $\lim_{x\to -\infty} \frac{x^2+3x^4-7x}{2x^3+2}$ $\longrightarrow$ ~~[illegible]~~ $-\infty$

(f) $\lim_{x\to \infty} \frac{3x^7-x^8+2}{3x^8-7}$ $\longrightarrow -\frac{1}{3}$

Question 4. *(10 marks)*
Find the derivative of the following functions.
(a) $f(x) = \cos(e^{2x})$

$$f'(x) = -\sin(e^{2x}) \cdot e^{2x} \cdot 2$$

(b) $f(x) = \sqrt[12]{x\sin x}$ $f'(x) = \frac{1}{12}(x\sin x)^{-11/12}\left[\sin x + x\cos x\right]$

(c) $f(x) = \ln(x^2 - \sin^4 x)$ $f'(x) = \frac{1}{x^2 - \sin^4 y}(2x - 4\sin^3 y \cos x)$

(d) $h(x) = x^2 + 3x - \frac{1}{\sqrt{x}}$ $h'(x) = 2x + 3 + \frac{1}{2}x^{-3/2}$

(e) $g(x) = \sin(e^{\cos x})$ $g'(x) = \cos(e^{\cos x}) \cdot e^{\cos x} \cdot (-\sin y)$

Question 5. *(3.5 marks)*
Find the equation of the tangent line to the curve $f(x) = \ln(x^x) - 3x$ at the point (1,0).

$f(x) = x\ln x - 3x$

SLOPE: $f'(x) = \ln x + x\left(\frac{1}{x}\right) - 3$

$= \ln x + 1 - 3$

$= \ln x - 2$

$f'(1) = \ln(1) - 2 = -2$

$y = -2x + b$

$0 = -2(1) + b$

$b = 2$

$\boxed{y = -2x + 2}$

Question 6. *(3 marks)*
Find the derivative of $g(t) = \sqrt{t}(e^{-t})\cos t$

$g(t) = \dfrac{t^{1/2}\cos t}{e^t}$

$g'(t) = \dfrac{\left[\frac{1}{2}t^{-1/2}\cos t - (\sin t)t^{1/2}\right]e^t - e^t(t^{1/2}\cos t)}{(e^t)^2}$

Question 7. *(3.5 marks)*
Find the value of the constant a if the slope of the tangent line to the curve $y = -3ax^2 + 3x + 2$ at $x = -1$ is equal to 6.

$y' = -6ax + 3$

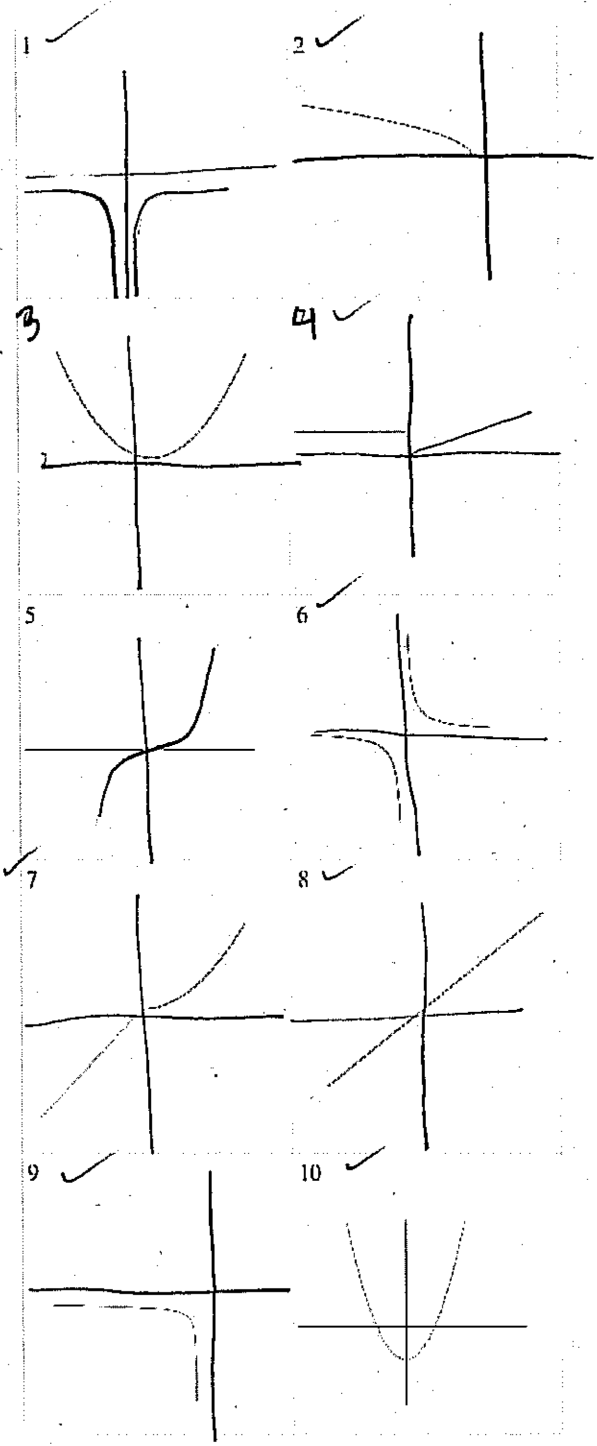
at $x = -1$ $\quad y' = 6$

$6 = -6a(-1) + 3$

$3 = 6a$

$a = \frac{1}{2}$

Questions 8. *(5 marks)*
Match the graph of each function with the graph of its derivative. For each function/derivative pair, make sure to indicate which is the derivative.



FUNCTION	derivative
7	4
10	8
2	9
6	1
5	3

Bonus *(5 marks)*

(a) Explain in your own words what the following formula represents:

$$\lim_{\Delta x \to 0} \frac{\sqrt{1+\Delta x}-\sqrt{1}}{\Delta x}$$

SLOPE OF TANGENT LINE TO THE curve $y=\sqrt{x}$
AT $x=1$

(b) Explain in your own words what the following formula represents:

$$\lim_{\Delta x \to 0} \frac{(3+\Delta x)^2-3^2}{\Delta x}$$

Slope OF TANGENT line TO THE curve AT $x=3$

(c) Use the definition of the derivative (not the rules) to compute the derivative of $f(x)=x^2+5x$ at the point $(1,6)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 + 5(x+\Delta x) - x^2 - 5x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x \overset{(+\Delta x^2)}{} + \cancel{5x} + 5\Delta x - \cancel{x^2} - \cancel{5x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + 5\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 2x + \Delta x + 5$$

$$= 2x+5$$

AT $x=1$

$$f'(x) = 7$$