NAME: SOLUTIONS

TEST 3

DAWSON COLLEGE

NYA-Electrotech Section 6 - Calculus 1

Instructor: E. Richer

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This test is marked out of **50 points**

Question 1. *(5 marks)*

Find $f(x)$ given the following information:

- $f'(x) = \sin x + 2x - 1$
- $f(\frac{\pi}{2}) = 0$

$$f(x) = \int \sin x + 2x - 1$$

$$= -\cos x + x^2 - x + C$$

$$f(\pi/2) = 0$$

$$\Longrightarrow 0 = -\cos(\pi/2) + (\pi/2)^2 - \pi/2 + C$$

$$0 = \pi^2/4 - \pi/2 + C$$

$$C = \pi/2 - \pi^2/4$$

$$\boxed{f(x) = -\cos x + x^2 - x + \pi/2 - \pi^2/4}$$

Question 2. *(10 marks)*

(a) $\int 3x^3 - 3\sqrt{x}\, dx$

$$= \frac{3x^4}{4} - \frac{3x^{3/2}}{3/2} + C$$

$$= \boxed{\tfrac{3}{4}x^4 - 2x^{3/2} + C}$$

(b) $\int \frac{2}{x^2} + \frac{6}{x}\, dx$

$$= \int 2x^{-2} + \frac{6}{x}\, dx$$

$$= \frac{2x^{-1}}{-1} + 6\ln x + C = \boxed{-\frac{2}{x} + 6\ln x + C}$$

(c) $\int \frac{1}{x} - \sec^2 x\, dx$

$$= \boxed{\ln x - \tan x + C}$$

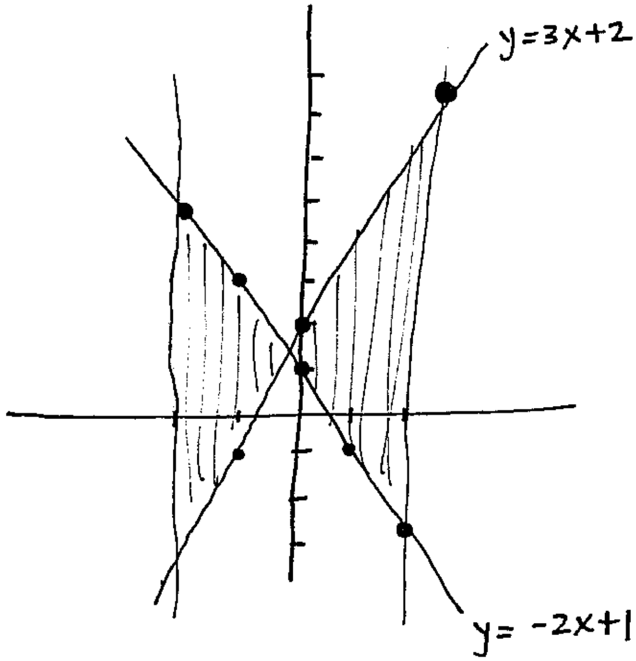
(d) $\int 2e^x + \pi\, dx$

$$= \boxed{2e^x + \pi x + C}$$

(e) $\int (\ln 3)3^x - \cos x\, dx$

$$= \boxed{3^x - \sin x + C}$$

Question 3. *(5 marks)*
Find the area bounded by the lines $2y = -4x + 2$ and $3y = 9x + 6$ on the interval $-2 \leq x \leq 2$. Include a sketch illustrating the area that you are calculating.



Line 1: $2y = -4x + 2$
$y = -2x + 1$

Line 2: $3y = 9x + 6$
$y = 3x + 2$

Intersection: $-2x + 1 = 3x + 2$
$-5x = 1$
$x = -\frac{1}{5}$

$$\text{Area} = \int_{-2}^{-1/5} (-2x+1) - (3x+2)\,dx + \int_{-1/5}^{2} (3x+2) - (-2x+1)\,dx$$

$$= \int_{-2}^{-1/5} -5x - 1\,dx + \int_{-1/5}^{2} 5x + 1\,dx$$

$$= \left(-5\frac{x^2}{2} - x\right)\Big|_{-2}^{-1/5} + \left(5\frac{x^2}{2} + x\right)\Big|_{-1/5}^{2}$$

$$= \left[\left(-\frac{1}{10} + \frac{1}{5}\right) - (-10 + 2)\right] + \left[(10+2) - \left(\frac{1}{10} - \frac{1}{5}\right)\right]$$

$$= \left(\frac{1}{10} + 8\right) + \left(12 + \frac{1}{10}\right)$$

$$= 20 + \frac{2}{10} = 20 + \frac{1}{5} = \boxed{20.2}$$

Question 4. *(5 marks)*
A current $i = \frac{2t}{\sqrt{t^2+1}}$ (in A) is sent through an electric dryer circuit containing a previously uncharged 4.0 μF capacitor. How long does it take for the capacitor voltage to reach 60V?

$$V = \frac{1}{C} q$$

$$q = \int i \, dt$$

$$= \int \frac{2t}{\sqrt{t^2+1}} dt$$

$$= \int \frac{1}{\sqrt{U}} du \qquad U = t^2+1 \qquad du = 2t \, dt$$

$$= \int U^{-1/2} du$$

$$= \frac{U^{1/2}}{1/2} + C_1$$

$$= 2\sqrt{U} + C_1 = 2\sqrt{t^2+1} + C_1$$

$$V = \frac{1}{4\mu}\left(2\sqrt{t^2+1} + C_1\right)$$

$$0 = \frac{1}{4\mu}\left[2 + C_1\right]$$

$$C_1 = -2$$

$$V = \frac{1}{4\mu}\left(2\sqrt{t^2+1} - 2\right)$$

$$60 = \frac{1}{4\mu}\left(2\sqrt{t^2+1} - 2\right)$$

$$240\mu = \left(2\sqrt{t^2+1} - 2\right)$$

$$\frac{240\mu + 2}{2} = \sqrt{t^2+1}$$

$$\left(\frac{240\mu+2}{2}\right)^2 = t^2 + 1$$

$$t^2 = \left(\frac{240\mu+2}{2}\right)^2 - 1$$

$$t = 0.01549$$

$$= \boxed{15.49 \text{ ms}}$$

Question 5. *(15 marks)*

Integrate each of the functions. *(5 marks each)*

(a) $\int \frac{\sqrt{\ln x}}{2x}\,dx$

$U = \ln x$

$du = \frac{1}{x}\,dx$

$\int \frac{\sqrt{u}}{2}\,du$

$= \frac{1}{2}\frac{U^{3/2}}{3/2} + C$

$= \frac{1}{3}U^{3/2} + C$

$= \boxed{\frac{1}{3}(\ln x)^{3/2} + C}$

(b) $\int (\tan^6 x)(\sec^2 x)\,dx$

$U = \tan x$

$du = \sec^2 x\,dx$

$\int U^6\,du$

$= \frac{U^7}{7} + C$

$= \boxed{\frac{1}{7}\tan^7 x + C}$

(c) $\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$

$U = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} \cdot dx$

$\int e^u\,du$

$= e^u + C$

$= \boxed{e^{\sqrt{x}} + C}$

Question 6. *(10 marks)*
Sketch the graph of $f(x)=x^4-2x^3+x^2$. Find and clearly identify on the sketch the following:

(a) The x and y intercepts

$$Y = x^4 - 2x^3 + x^2$$
$$= x^2(x^2-2x+1)$$
$$= x^2(x-1)^2$$

$$\boxed{\text{INTERCEPTS } (0,0)\ (1,0)}$$

(b) The behavior of the function as x tends to $\pm\infty$

$$\lim_{x\to\infty} f(x) \longrightarrow +\infty$$

$$\lim_{x\to-\infty} f(x) \longrightarrow +\infty$$

(c) The intervals where $f(x)$ is increasing/decreasing and any relative maxima or minima.

$$f'(x) = 4x^3-6x^2+2x$$
$$= 2x(2x^2-3x+1)$$
$$= 2x(2x^2-2x-x+1)$$
$$= 2x(2x(x-1)-1(x-1))$$
$$= 2x(2x-1)(x-1)$$

$x=0 \quad x=\frac{1}{2} \quad x=1$

INTERVALS	$(-\infty,0)$	$(0,\frac{1}{2})$	$(\frac{1}{2},1)$	$(1,\infty)$
test pt	-1	$\frac{1}{4}$	$\frac{3}{4}$	2
Sign of f'	$-$	$+$	$-$	$+$
	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$

min AT $(0,0)$
MAX AT $x=\frac{1}{2}$ $\quad y = (\frac{1}{2})^4 - 2(\frac{1}{2})^3 + (\frac{1}{2})^2 = \frac{1}{16} - \frac{1}{4} + \frac{1}{4} = \frac{1}{16}$ $\quad (\frac{1}{2}, \frac{1}{16})$

min AT $x=+1$ $\quad y=0$

(d) The intervals where $f(x)$ is concave up/down and any points of inflection

$$f''(x) = 12x^2 - 12x + 2 = 2(6x^2 - 6x + 1)$$

$$x = \frac{6 \pm \sqrt{36 - 4(6)(1)}}{12} = \frac{6 \pm \sqrt{12}}{12} = \frac{6 \pm 2\sqrt{3}}{12} = \frac{1}{6}(3 \pm \sqrt{3})$$

$x \approx 0.78$ & $x \approx 0.21$

INTERVALS	$(-\infty, 0.21)$	$(0.21, 0.78)$	$(0.78, \infty)$
TEST	0	0.5	1
Sign of f''	+	—	+
	$\cup$	$\cap$	$\cup$

INFLECTION PTS AT $x = 0.21$
$x = 0.78$

SKETCH OF $f(x) = x^4 - 2x^3 + x^2$