The Fundamental Theorem of Calculus

Recall how the derivative is closely related to the area problem. We will now see how the antiderivative is related to the area problem.

Theorem: The Fundamental Theorem of Calculus (Evaluation Theorem)
Let $f(x)$ be a continuous function on $[a,b]$ and $F(x)$ any antiderivative of $f(x)$, that is, $F'(x) = f(x)$. Then

Notation:

Example: Let $f(x) = 1 + x$.

(a) Find $\displaystyle\int_1^3 f(x)\,dx$ using geometry.

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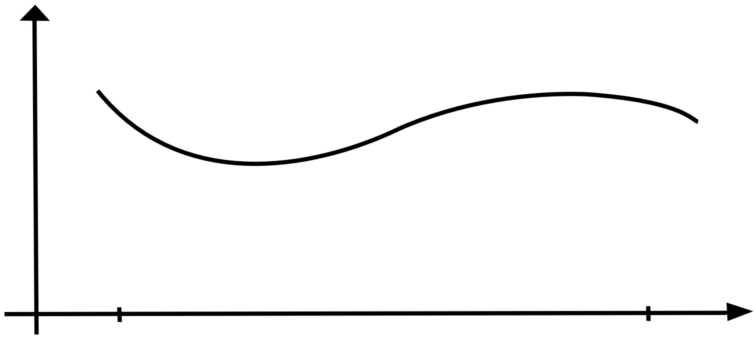
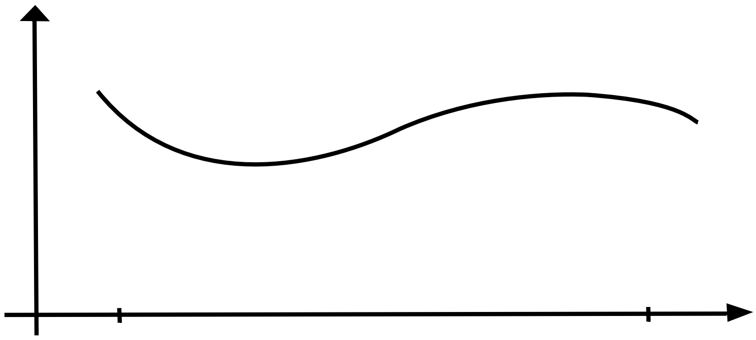
(b) Find $\int_1^3 f(x)\,dx$ using the definition of the definite integral.

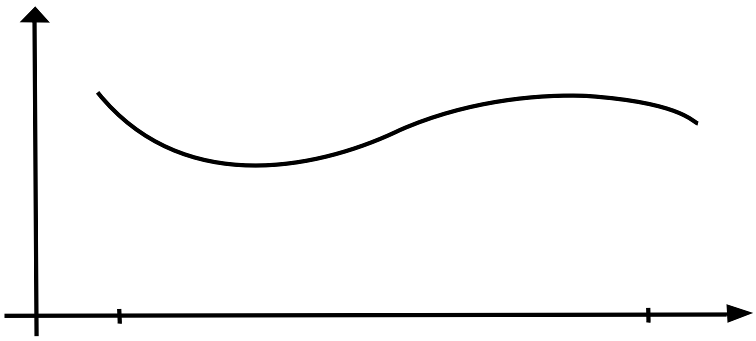
(c) Find $\int_1^3 f(x)\,dx$ using the FTC (evaluation theorem).

Example: Find the area underneath the curve $f(x) = x^2$ on the interval $[0, 1]$.

But why does FTC work?







Example: Evaluate the following integral:

1) $\int_0^2 (x+e^x)\,dx$

2) $\int_{-1}^0 (4-x)\,dx$

3) $\int_1^4 \frac{2x^3-x^2+2}{x^2}\,dx$

4) $\int_0^1 \sqrt{2x}(\sqrt{x}+\sqrt{2}\,dx$

5) $\int_{\pi/8}^{\pi/4} \cot 2x\,dx$

6) Find the area under $f(x)=x+\sin x$ from $x=\pi/4$ to $x=\pi/2$