Dawson College: **Calculus I**: 201-NYA-05-S4: Winter 2012

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Quiz 11

This quiz is graded out of 15 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.5 #14 *(5 marks)* For its beef stew, Betty Moore Company uses aluminum containers that have the form of right circular cylinders. Find the radius and height of a container if it has a capacity of $36in^3$ and is constructed using the least amount of metal.

Volume:

$V = \pi r^2 h$

① $36 = \pi r^2 h \iff h = \dfrac{36}{\pi r^2}$

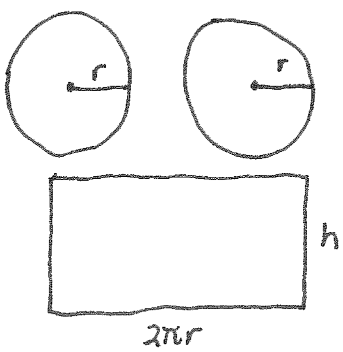
② $SA = 2\pi r^2 + 2\pi r h$

sub ② into ①

$SA = 2\pi r^2 + 2\pi r \dfrac{36}{\pi r^2}$

$= 2\pi r^2 + \dfrac{72}{r}$

Surface Area:



$SA' = 4\pi r - \dfrac{72}{r^2}$

$0 = SA'$

$0 = 4\pi r - \dfrac{72}{r^2}$

$\dfrac{72}{r^2} = 4\pi r$

$72 = 4\pi r^3$

$\dfrac{18}{\pi} = r^3$

$\sqrt[3]{\dfrac{18}{\pi}} = r$

$SA'' = 4\pi + \dfrac{128}{r^3}$

$SA''\left(\sqrt[3]{\dfrac{18}{\pi}}\right) = 4\pi + \dfrac{128}{\left(\sqrt[3]{\frac{18}{\pi}}\right)^3} > 0$

$\therefore r = \sqrt[3]{\dfrac{18}{\pi}}$ is a minimum

$\therefore h = \dfrac{36}{\pi\left(\sqrt[3]{\frac{18}{\pi}}\right)^2}$

Question 2. §4.4 #61 *(5 marks)* The quantity demanded each month of the Sicard wristwatch is related to the unit price by the equation

$$p = \frac{50}{0.01x^2+1} \quad (0 \le x \le 20)$$

where p is measured in dollars and x is measured in units of a thousand. To yield a maximum revenue, how many watches must be sold?

$R(x) = xp$

$= x\left(\dfrac{50}{0.01x^2+1}\right)$

$= \dfrac{50x}{0.01x^2+1}$

$R'(x) = \dfrac{50(0.01x^2+1) - 50x(0.02x)}{(0.01x^2+1)^2}$

$= \dfrac{0.5x^2 + 50 - x^2}{(0.01x^2+1)^2}$

$= \dfrac{-0.5x^2+50}{(0.01x^2+1)^2}$

$0 = R'(x)$

$0 = \dfrac{-0.5x^2+50}{(0.01x^2+1)^2}$

$50 = 0.5x^2$

$100 = x^2$

$\pm 10 = x$

$\therefore x = 10$ and $x = -10$ not valid

$R(0) = \dfrac{50(0)}{0.01(0)^2+1} = 0$

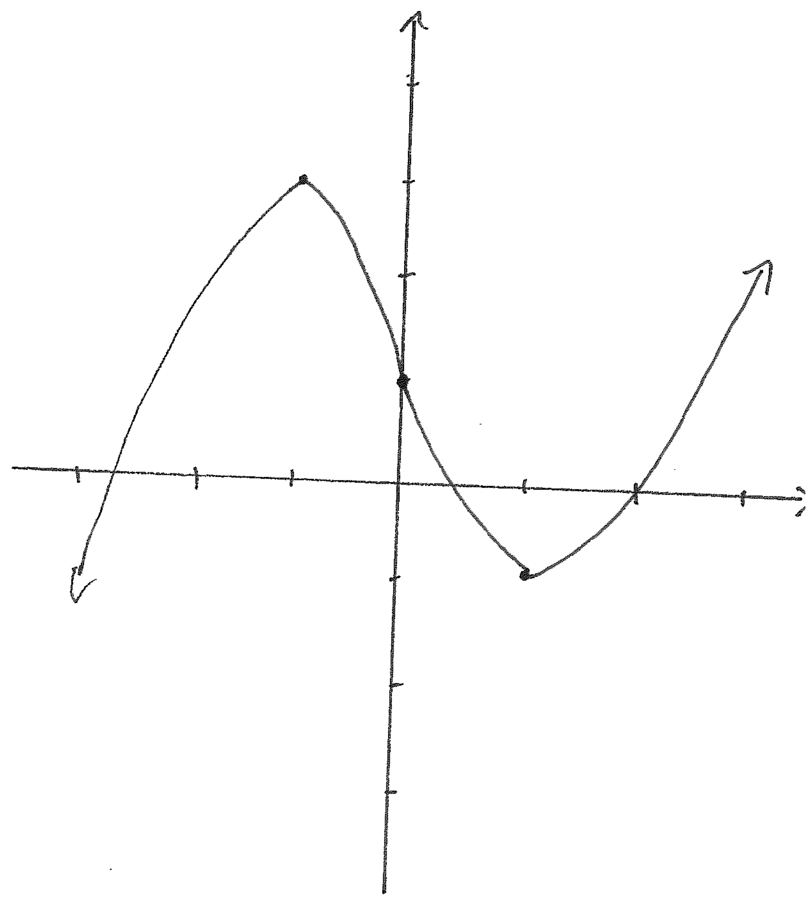
$R(10) = \dfrac{50(10)}{0.01(10)^2+1} = \dfrac{500}{1+1} = 250$

$R(20) = \dfrac{50(20)}{0.01(20)^2+1} = \dfrac{1000}{4+1} = 200.$

$\therefore$ max revenue at $x = 10$.

Question 3. §4.3 #39 *(5 marks)* Sketch the graph of the function:

$$h(x) = x^3 - 3x + 1$$



X-int: too difficult

y-int: (0, 1)

critical points:

$h'(x) = 3x^2 - 3$

So,
$0 = 3x^2 - 3$
$3 = 3x^2$
$1 = x^2$
$\pm 1 = x$

Domain of h(x): $\mathbb{R}$

Inc./dec. intervals:

	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
p	-2	0	2
h'(p)	+	−	+
inc/dec	↗	↘	↗

∴ local max at $x = -1$
$y = h(-1) = 3$

∴ local min at $x = 1$
$y = h(1) = -1$

concavity and inflection points:

$h''(x) = 6x$

So,
$0 = 6x$
$0 = x$

	$(-\infty, 0)$	$(0, \infty)$
p	-1	1
h''(p)	−	+
concavity	∩	∪

∴ inflection point
at $x = 0$
$y = 1$